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ABSTRACT

As a supplement to Project Physics Unit 1, 21 articles are presented in this reader. Concepts of motion are discussed under headings: motion, motion in words, representation of movement, introducing vectors, Galileo's discussion of projectile motion, Newton's laws of dynamics, the dynamics of a golf club, report on Tait's lecture on force, and bad physics in athletic measurements. The remaining excerpts and book passages are related to the value of science, close reasoning, scientific method, problem-solving techniques, advice on planning a career in sciences, right size of animals, scientific revolution, effects of the rise of physics in the age of Galileo and Newton, fun in space, vision of our age, making of a scientist, and chart of the future. Also included are illustrations for explanation purposes. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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Project Physics **Reader** 1

An Introduction to Physics

Concepts of Motion

This document is a preliminary version of only one of many instructional materials being developed by Harvard Project Physics. Like all existing Project materials—text units, laboratory experiments, teachers' guides, etc.—it is based on an earlier draft used in cooperating schools. This version of the Reader, issued in six separate booklets, was developed under the direction of Professor Alfred M. Bork of Reed College, Portland, Oregon.

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Project Physics **Reader**

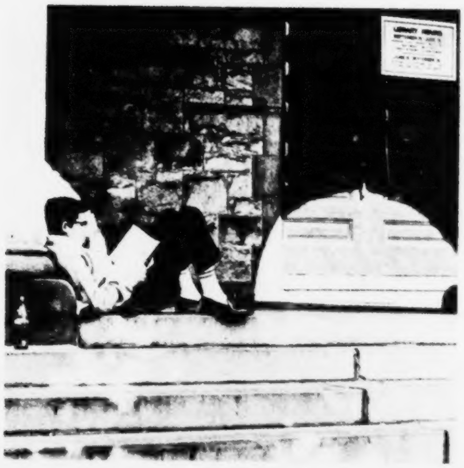
An Introduction to Physics

1

Concepts of Motion

Preliminary Version

1967-68



This is not a physics textbook. Rather, it is a physics reader, a collection of some of the best articles and book passages on physics. A few are on historic events in science, others contain some particularly memorable description of what physicists do; still others deal with philosophy of science, or with the impact of scientific thought on the imagination of the artist.

There are old and new classics, and also some little-known publications; many have been suggested for inclusion because some teacher or physicist remembered an article with particular fondness. The majority of articles is not drawn from scientific papers of historic importance themselves, because material from many of these is readily available, either as quotations in the Project Physics text or in special collections.

This collection is meant for your browsing. If you follow your own reading interests, chances are good that you will find here many pages that convey the joy these authors have in their work and the excitement of their ideas. If you want to follow up on interesting excerpts, the source list at the end of the reader will guide you for further reading.

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An outstanding contemporary theoretical physicist reminisces informally about science and its role in society. Feynman stresses the importance in science, and elsewhere, of admitting that one does not know all the answers.

1

The Value of Science

Richard P. Feynman

1958

From time to time, people suggest to me that scientists ought to give more consideration to social problems—especially that they should be more responsible in considering the impact of science upon society. This same suggestion must be made to many other scientists, and it seems to be generally believed that if the scientists would only look at these very difficult social problems and not spend so much time fooling with the less vital scientific ones, great success would come of it.

It seems to me that we do think about these problems from time to time, but we don't put full-time effort into them—the reason being that we know we don't have any magic formula for solving problems, that social problems are very much harder than scientific ones, and that we usually don't get anywhere when we do think about them.

I believe that a scientist looking at nonscientific problems is just as dumb as the next guy—and when he talks about a nonscientific matter, he will sound as naive as anyone untrained in the matter. Since the question of the value of science is not a scientific subject, this discussion is dedicated to proving my point—by example.

The first way in which science is of value is familiar to everyone. It is that scientific knowledge enables us to do all kinds of things and to make all kinds of things. Of course if we make good things, it is not only to the credit of science; it is also to the credit of the moral choice which led us to good work. Scientific knowledge is an enabling power to do either good or bad—but it does not carry instructions on how to use it. Such power has evident value—even though the power may be negated by what one does.

I learned a way of expressing this common human problem on a trip to Honolulu. In a Buddhist temple there, the man in charge explained a little bit about the Buddhist religion for tourists, and then ended his talk by telling them he had something to say to them that they would *never* forget—and I have never forgotten it. It was a proverb of the Buddhist religion:

"To every man is given the key to the gates of heaven; the same key opens the gates of hell."

What then, is the value of the key to heaven? It is true that if we lack clear instructions that determine which is the gate to heaven and which the gate to hell, the key may be a dangerous object to use, but it obviously has value. How can we enter heaven without it?

The instructions, also, would be of no value without the key. So it is evident that, in spite of the fact that science could produce enormous horror in the world, it is of value because it *can* produce *something*.

Another value of science is the fun called intellectual enjoyment which some people get from reading and learning and thinking about it, and which others get from working in it. This is a very real and important point and one which is not considered enough by those who tell us it is our social responsibility to reflect on the impact of science on society.

Is this mere personal enjoyment of value to society as a whole? No! But it is also a responsibility to consider the value of society itself. Is it, in the last analysis, to arrange things so that people can enjoy things? If so, the enjoyment of science is as important as anything else.

But I would like *not* to underestimate the value of the world view which is the result of scientific effort. We have been led to imagine all sorts of things infinitely more marvelous than the imaginings of poets and dreamers of the past. It shows that the imagination of nature is far, far greater than the imagination of man. For instance, how much more remarkable it is for us all to be stuck—half of us upside down—by a mysterious attraction, to a spinning ball that has been swinging in space for billions of years, than to be carried on the back of an elephant supported on a tortoise swimming in a bottomless sea.

I have thought about these things so many times alone that I hope you will excuse me if I remind you of some thoughts that I am sure you have all had—or this type of thought—which no one could ever have had in the past, because people then didn't have the information we have about the world today.

For instance, I stand at the seashore, alone, and start to think. There are the rushing waves . . . mountains of molecules, each stupidly minding its own business . . . trillions apart . . . yet forming white surf in unison.

Ages on ages . . . before any eyes could see . . . year after year . . . thunderously pounding the shore as now. For whom, for what? . . . on a dead planet, with no life to entertain.

Never at rest . . . tortured by energy . . . wasted prodigiously by the sun . . . poured into space. A mite makes the sea roar.

Deep in the sea, all molecules repeat the patterns of one another till complex new ones are formed. They make others like themselves . . . and a new dance starts.

Growing in size and complexity . . . living things, masses of atoms, DNA, protein . . . dancing a pattern ever more intricate.

Out of the cradle onto the dry land . . . here it is standing . . . atoms with consciousness . . . matter with curiosity.

Stands at the sea . . . wonders at wondering . . . I . . . a universe of atoms . . . an atom in the universe.

THE GRAND ADVENTURE

The same thrill, the same awe and mystery, come again and again when we look at any problem deeply enough. With more knowledge comes deeper, more wonderful mystery, luring one on to penetrate deeper still. Never concerned that the answer may prove disappointing, but with pleasure and confidence we turn over each new stone to find unimagined strangeness leading on to more wonderful questions and mysteries—certainly a grand adventure!

It is true that few unscientific people have this particular type of religious experience. Our poets do not write about it; our artists do not try to portray this remarkable thing. I don't know why. Is nobody inspired by our present picture of the universe? The value of science remains unsung by singers, so you are reduced to hearing—not a song or a poem, but an evening lecture about it. This is not yet a scientific age.

Perhaps one of the reasons is that you have to know how to read the music. For instance, the scientific article says, perhaps, something like this: "The radioactive phosphorus content of the cerebrum of the rat decreases to one-half in a period of two weeks." Now, what does that mean?

It means that phosphorus that is in the brain of a rat (and also in mine, and yours) is not the same phosphorus as it was two weeks ago, but that all of the atoms that are in the brain are being replaced, and the ones that were there before have gone away.

So what is this mind, what are these atoms with consciousness? Last week's potatoes! That is what now can *remember* what was going on in my mind a year ago—a mind which has long ago been replaced.

That is what it means when one discovers how long it takes for the atoms of the brain to be replaced by other atoms, to note that the thing which I call my individuality is only a pattern or dance. The atoms come into my brain, dance a dance, then go out; always new atoms but always doing the same dance, remembering what the dance was yesterday.

THE REMARKABLE IDEA

When we read about this in the newspaper, it says, "The scientist says that this discovery may have importance in the cure of cancer." The paper is only interested in the use of the idea, not the idea itself. Hardly anyone can understand the importance of an idea, it is so remarkable. Except that, possibly, some children catch on. And when a child catches on to an idea like that, we have a scientist. These ideas do filter down (in spite of all the conversation about TV replacing thinking), and lots of kids get the spirit—and when they have the spirit you have a scientist. It's too late for them to get the spirit when they are in our universities, so we must attempt to explain these ideas to children.

I would now like to turn to a third value that science has. It is a little more indirect, but not much. The scientist has a lot of experience with ignorance and doubt and uncertainty, and this experience is of very great importance, I think. When a scientist doesn't know the answer to a problem, he is ignorant. When he has a hunch as to what the result is, he is uncertain. And when he is pretty darn sure of what the result is going to be, he is in some doubt. We have found it of paramount importance that in order to progress we must recognize the ignorance and leave room for doubt. Scientific knowledge is a body of statements of varying degrees of certainty—some most unsure, some nearly sure, none *absolutely* certain.

Now, we scientists are used to this, and we take it for granted that it is perfectly consistent to be unsure—that it is possible to live and *not* know. But I don't know whether everyone realizes that this is true. Our freedom to doubt was born of a struggle against authority in the early days of science. It was a very deep and strong struggle. Permit us to question—to doubt, that's all—not to be sure. And I think it is important that we do not forget the importance of this struggle and thus perhaps lose what we have gained. Here lies a responsibility to society.

We are all sad when we think of the wondrous potentialities human beings seem to have, as contrasted with their small accomplishments. Again and again people have thought that we could do much better. They of the past saw in the nightmare of their times a dream for the future. We, of their future, see that their dreams, in certain ways surpassed, have in many ways remained dreams. The hopes for the future today are, in good share, those of yesterday.

EDUCATION, FOR GOOD AND EVIL

Once some thought that the possibilities people had were not developed because most of those people were ignorant. With education universal, could all men be Voltaires? Bad can be taught at least as efficiently as good. Education is a strong force, but for either good or evil.

Communications between nations must promote understanding: so went another dream. But the machines of communication can be channeled or choked. What is communicated can be truth or lie. Communication is a strong force also, but for either good or bad.

The applied sciences should free men of material problems at least. Medicine controls diseases. And the record here seems all to the good. Yet there are men patiently working to create great plagues and poisons. They are to be used in warfare tomorrow.

Nearly everybody dislikes war. Our dream today is peace. In peace, man can develop best the enormous possibilities he seems to have. But maybe future men will find that peace, too, can be good and bad. Perhaps peaceful men will drink out of boredom. Then perhaps drink will become the great problem which seems to keep man from getting all he thinks he should out of his abilities.

Clearly, peace is a great force, as is sobriety, as are material power, communication, education, honesty and the ideals of many dreamers.

We have more of these forces to control than did the ancients. And maybe we are doing a little better than most of them could do. But what we ought to be able to do seems gigantic compared with our confused accomplishments.

Why is this? Why can't we conquer ourselves?

Because we find that even great forces and abilities do not seem to carry with them clear instructions on how to use them.

As an example, the great accumulation of understanding as to how the physical world behaves only convinces one that this behavior seems to have a kind of meaninglessness. The sciences do not directly teach good and bad.

Through all ages men have tried to fathom the meaning of life. They have realized that if some direction or meaning could be given to our actions, great human forces would be unleashed. So, very many answers must have been given to the question of the meaning of it all. But they have been of all different sorts, and the proponents of one answer have looked with horror at the actions of the believers in another. Horror, because from a disagreeing point of view all the great potentialities of the race were being channeled into a false and confining blind alley. In fact, it is from the history of the enormous monstrosities created by false belief that philosophers have realized the apparently infinite and wondrous capacities of human beings. The dream is to find the open channel.

What, then, is the meaning of it all? What can we say to dispel the mystery of existence?

If we take everything into account, not only what the ancients knew, but all of what we know today that they didn't know, then I think that we must frankly admit that *we do not know.*

But, in admitting this, we have probably found the open channel.

This is not a new idea; this is the idea of the age of reason. This is the philosophy that guided the men who made the democracy that we live under. The idea that no one really knew how to run a government led to the idea that we should arrange a system by which new ideas could be developed, tried out, tossed out, more new ideas brought in; a trial and error system. This method was a result of the fact that science was already showing itself to be a successful venture at the end of the 18th century. Even then it was clear to socially-minded people that the openness of the possibilities was an opportunity, and that doubt and discussion were essential to progress into the unknown. If we want to solve a problem that we have never solved before, we must leave the door to the unknown ajar.

OUR RESPONSIBILITY AS SCIENTISTS

We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. There are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions and pass them on. It is our responsibility to leave the men of the future a free hand. In the impetuous youth of humanity, we can make grave errors that can stunt our growth for a long time. This we will do if we say we have the answers now, so young and ignorant; if we suppress all discussion, all criticism, saying, "This is it, boys, man is saved!" and thus doom man for a long time to the chains of authority, confined to the limits of our present imagination. It has been done so many times before.

It is our responsibility as scientists, knowing the great progress and great value of a satisfactory philosophy of ignorance, the great progress that is the fruit of freedom of thought, to proclaim the value of this freedom, to teach how doubt is not to be feared but welcomed and discussed, and to demand this freedom as our duty to all coming generations.

This chapter from a science fiction novel by a present-day astronomer offers some non-fiction insight into the way the scientist works. Another chapter from this same novel is in the Unit 2 Reader.

2 Close Reasoning

Fred Hoyle

1957

It is curious in how great a degree human progress depends on the individual. Humans, numbered in thousands of millions, seem organised into an ant-like society. Yet this is not so. New ideas, the impetus of all development, come from individual people, not from corporations or states. New ideas, fragile as spring flowers, easily bruised by the tread of the multitude, may yet be cherished by the solitary wanderer.

Among the vast host that experienced the coming of the Cloud, none except Kingsley arrived at a coherent understanding of its real nature, none except Kingsley gave the reason for the visit of the Cloud to the solar system. His first bald statement was greeted with outright disbelief even by his fellow scientists—Alexandrov excepted.

Weichart was frank in his opinion.

"The whole idea is quite ridiculous," he said.

Marlowe shook his head.

"This comes of reading science fiction."

"No bloody fiction about Cloud coming straight for dam' Sun. No bloody fiction about Cloud stopping. No bloody fiction about ionisation," growled Alexandrov.

McNeil, the physician, was intrigued. The new development was more in his line than transmitters and aerials.

"I'd like to know, Chris, what you mean in this context by the word 'alive.' "

"Well, John, you know better than I do that the distinction between animate and inanimate is more a matter of verbal convenience than anything else. By and large, inanimate matter has a simple structure and comparatively simple properties. Animate or living matter on the other hand has a highly complicated structure and is capable of very involved behaviour. When I said the Cloud may be alive I meant that the material inside it may be organised in an intricate fashion, so that its behaviour and consequently the behaviour of the whole Cloud is far more complex than we previously supposed."

"Isn't there an element of tautology there?"—from Weichart.

"I said that words such as 'animate' and 'inanimate' are only verbal conveniences. If they're pushed too far they do appear tautological. In more scientific terms I expect the chemistry of the interior of the Cloud to be extremely complicated—complicated molecules, complicated structures built out of molecules, complicated nervous activity. In short I think the Cloud has a brain."

"A dam' straightforward conclusion," nodded Alexandrov.

When the laugh had subsided, Marlowe turned to Kingsley.

"Well, Chris, we know what you mean, at any rate we know near enough. Now let's have your argument. Take your time. Let's have it point by point, and it'd better be good."

"Very well then, here goes. Point number one, the temperature inside the Cloud is suited to the formation of highly complicated molecules."

"Right! First point to you. In fact, the temperature is perhaps a little more favourable than it is here on the Earth."

"Second point, conditions are favourable to the formation of extensive structures built out of complicated molecules."

"Why should that be so?" asked Yvette Hedelfort.

"Adhesion on the surface of solid particles. The density inside the Cloud is so high that quite large lumps of solid material—probably mostly ordinary ice—are almost certainly to be found inside it. I suggest that the complicated molecules get together when they happen to stick to the surfaces of these lumps."

"A very good point, Chris," agreed Marlowe.

"Sorry, I don't pass this round." McNeil was shaking his head. "You talk of complicated molecules being built up by sticking together on the surface of solid bodies. Well, it won't do. The molecules out of which living material is made contain large stores of internal energy. Indeed, the processes of life depend on this internal energy. The trouble with your sticking together is that you don't get energy into the molecules that way."

Kingsley seemed unperturbed.

"And from what source do the molecules of living creatures here on the Earth get their internal supplies of energy?" he asked McNeil.

"Plants get it from sunlight, and animals get it from plants, or from other animals of course. So in the last analysis the energy always comes from the Sun."

"And where is the Cloud getting energy from now?"

The tables were turned. And as neither McNeil nor anyone else seemed disposed to argue, Kingsley went on:

"Let's accept John's argument. Let's suppose that my beast in the Cloud is built out of the same sort of molecules that we are. Then the light from some star is required in order that the molecules be formed. Well, of course starlight is available far out in the space between the stars, but it's very feeble. So to get a really strong supply of light the beast would need to approach close to some star. And that's just what the beast has done!"

Marlowe became excited.

"My God, that ties three things together, straight away. The need for sunlight, number one. The Cloud making a bee-line for the Sun, number two. The Cloud stopping when it reached the Sun, number three. Very good, Chris."

"It is a very good beginning, yes, but it leaves some things obscure," Yvette Hedelfort remarked. "I do not see," she went on, "how it was that the Cloud came to be out in space. If it has need of sunlight or starlight, surely it would stay always around one star. Do you suppose that this beast of yours has just been born somewhere out in space and has now come to attach itself to our Sun?"

"And while you're about it, Chris, will you explain how your friend the beast controls its supplies of energy? How did it manage to fire off those blobs of gas with such fantastic speed when it was slowing down?" asked Leicester.

"One question at a time! I'll take Harry's first, because it's probably easier. We tried to explain the expulsion of those blobs of gas in terms of magnetic fields, and the explanation simply didn't work. The trouble was that the required fields would be so intense that they'd simply burst the whole Cloud apart. Stated somewhat differently, we couldn't find any way in which large quantities of energy could be localised through a magnetic agency in comparatively small regions. But let's now look at the problem from this new point of view. Let's begin by asking what method we ourselves would use to produce intense local concentrations of energy."

"Explosions!" gasped Barnett.

"That's right, explosions, either by nuclear fission, or more probably by nuclear fusion. There's no shortage of hydrogen in this Cloud."

"Are you being serious, Chris?"

"Of course I'm being serious. If I'm right in supposing that some beast inhabits the Cloud, then why shouldn't he be at least as intelligent as we are?"

"There's the slight difficulty of radioactive products. Wouldn't these be extremely deleterious to living material?" asked McNeil.

"If they could get at the living material, certainly they would. But although it isn't possible to produce explosions with magnetic fields, it is possible to prevent two samples of material mixing with each other. I imagine that the beast orders the material of the Cloud magnetically, that by means of magnetic fields he can move samples of material wherever he wants inside the Cloud. I imagine that he takes very good care to keep the radioactive gas well separated from the living material—remember I'm using the term 'living' for verbal convenience. I'm not going to be drawn into a philosophical argument about it."

"You know, Kingsley," said Weichart, "this is going far better than I thought it would. What I suppose you would say is that whereas basically we assemble materials with our hands, or with the aid of machines that we have made with our hands, the beast assembles materials with the aid of magnetic energy."

"That's the general idea. And I must add that the beast seems to me to have far the better of it. For one thing he's got vastly more energy to play with than we have."

"My God, I should think so, billions of times more, at the very least," said Marlowe. "It's beginning to look, Chris, as if you're winning this argument. But we objectors over here in this corner are pinning our faith to Yvette's question. It seems to me a very good one. What can you offer in answer to it?"

"It is a very good question, Geoff, and I don't know that I can give a really convincing answer. The sort of idea I've got is that perhaps the beast can't stay for very long in the close proximity of a star. Perhaps he comes in periodically to some star or other, builds his molecules, which form his food supply as it were, and then pushes off again. Perhaps he does this time and time again."

"But why shouldn't the beast be able to stay permanently near a star?"

"Well, an ordinary common or garden cloud, a beastless cloud, if it were permanently near a star, would gradually condense into a compact body, or into a number of compact bodies. Indeed, as we all know, our Earth probably condensed at one time from just such a cloud. Obviously our friend the beast would find it extremely embarrassing to have his protective Cloud condense into a planet. So equally obviously he'll decide to push off before there's any danger of that happening. And when he pushes off he'll take his Cloud with him."

"Have you any idea of how long that will be?" asked Parkinson.

"None at all. I suggest that the beast will push off when he's finished recharging his food supply. That might be a matter of weeks, months, years, millennia for all I know."

"Don't I detect a slight smell of rat in all this?" Barnett remarked.

"Possibly. I don't know how keen your sense of smell is, Bill. What's your trouble?"

"I've got lots of troubles. I should have thought that your remarks about condensing into a planet apply only to an inanimate cloud. If we grant that the Cloud is able to control the distribution of material within itself, then it could easily prevent condensation from taking place. After all, condensation must be a sort of stability process and I would have thought that quite a moderate degree of control on the part of your beast could prevent any condensation at all."

"There are two replies to that. One is that I believe the beast will lose his control if he stays too long near the Sun. If he stays too long, the magnetic field of the Sun will penetrate into the Cloud. Then the rotation of the Cloud round the Sun will twist up the magnetic field to blazes. All control would then be lost."

"My God, that's an excellent point."

"It is, isn't it? And here's another one. However different our beast is to life here on Earth, one point he must have in common with us. We must both obey the simple biological rules of selection and development. By that I mean that we can't suppose that the Cloud started off by containing a fully-fledged beast. It must have started with small beginnings, just as life here on Earth started with small beginnings. So to start with there would be no intricate control over the distribution of material in the Cloud. Hence if the Cloud had originally been situated close to a star, it could not have prevented condensation into a planet or into a number of planets."

"Then how do you visualise the early beginnings?"

"As something that happened far out in interstellar space. To begin with, life in the Cloud must have depended on the general radiation field of the stars. Even that would give it more radiation for molecule-building purposes than life on the Earth gets. Then I imagine that as intelligence developed it would be discovered that food supplies—i.e. molecule-building—could be enormously increased by moving in close to a star for a comparatively brief period. As I see it, the beast must be essentially a denizen of interstellar space. Now, Bill, have you any more troubles?"

"Well, yes, I've got another problem. Why can't the Cloud manufacture its own radiation? Why bother to come in close to a star? If it understands nuclear fusion to the point of producing gigantic explosions, why not use nuclear fusion for producing its supply of radiation?"

"To produce radiation in a controlled fashion requires a slow reactor, and of course that's just what a star is. The Sun is just a gigantic slow nuclear fusion reactor. To produce radiation on any real scale comparable with the Sun, the Cloud would have to make itself into a star. Then the beast would get roasted. It'd be much too hot inside."

"Even then I doubt whether a cloud of this mass could produce very much radiation," remarked Marlowe. "Its mass is much too small. According to the mass-luminosity relation it'd be down as compared with the Sun by a fantastic amount. No, you're barking up a wrong tree there, Bill."

"I've a question that I'd like to ask," said Parkinson. "Why do you always refer to your beast in the singular? Why shouldn't there be lots of little beasts in the Cloud?"

"I have a reason for that, but it'll take quite a while to explain."

"Well, it looks as if we're not going to get much sleep tonight, so you'd better carry on."

"Then let's start by supposing that the Cloud contains lots of little beasts instead of one big beast. I think you'll grant me that communication must have developed between the different individuals."

"Certainly."

"Then what form will the communication take?"

"You're supposed to be telling us, Chris."

"My question was purely rhetorical. I suggest that communication would be impossible by our methods. We communicate acoustically."

"You mean by talking. That's certainly your method all right, Chris," said Ann Halsey.

But the point was lost on Kingsley. He went on.

"Any attempt to use sound would be drowned by the enormous amount of background noise that must exist inside the Cloud. It would be far worse than trying to talk in a roaring gale. I think we can be pretty sure that communication would have to take place electrically."

"That seems fair enough."

"Good. Well, the next point is that by our standards the distances between the individuals would be very large, since the Cloud by our standards is enormously large. It would obviously be intolerable to rely on essentially D.C. methods over such distances."

"D.C. methods? Chris, will you please try to avoid jargon."

"Direct current."

"That explains it, I suppose!"

"Oh, the sort of thing we get on the telephone. Roughly speaking the difference between D.C. communication and A.C. communication is the difference between the telephone and radio."

Marlowe grinned at Ann Halsey.

"What Chris is trying to say in his inimitable manner is that communication must occur by radiative propagation."

"If you think that makes it clearer. . . ."

"Of course it's clear. Stop being obstructive, Ann. Radiative propagation occurs when we emit a light signal or a radio signal. It travels across space through a vacuum at a speed of 186,000 miles per second. Even at this speed it would still take about ten minutes for a signal to travel across the Cloud.

"My next point is that the volume of information that can be transmitted radiatively is enormously greater than the amount that we can communicate by ordinary sound. We've seen that with our pulsed radio transmitters. So if this Cloud contains separate individuals, the individuals must be able to communicate on a vastly more detailed scale than we can. What we can get across in an hour of talk they might get across in a hundredth of a second."

"Ah, I begin to see light," broke in McNeil. "If communication occurs on such a scale then it becomes somewhat doubtful whether we should talk any more of separate individuals!"

"You're home, John!"

"But *I'm* not home," said Parkinson.

"In vulgar parlance," said McNeil amiably, "what Chris is saying is that individuals in the Cloud, if there are any, must be highly telepathic, so telepathic that it becomes rather meaningless to regard them as being really separate from each other."

"Then why didn't he say so in the first place?"—from Ann Halsey.

"Because like most vulgar parlance, the word 'telepathy' doesn't really mean very much."

"Well, it certainly means a great deal more to me."

"And what does it mean to you, Ann?"

"It means conveying one's thoughts without talking, or of course without writing or winking or anything like that."

"In other words it means—if it means anything at all—communication by a non-acoustic medium."

"And that means using radiative propagation," chipped in Leicester.

"And radiative propagation means the use of alternating currents, not the direct currents and voltages we use in our brains."

"But I thought we were capable of some degree of telepathy," suggested Parkinson.

"Rubbish. Our brains simply don't work the right way for telepathy. Everything is based on D.C. voltages, and radiative transmission is impossible that way."

"I know this is rather a red herring, but I thought these extrasensory people had established some rather remarkable correlations," Parkinson persisted.

"Bloody bad science," growled Alexandrov. "Correlations obtained after experiments done is bloody bad. Only prediction in science."

"I don't follow."

"What Alexis means is that only predictions really count in science," explained Weichart. "That's the way Kingsley downed me an hour or two ago. It's no good doing a lot of experiments first and then discovering a lot of correlations afterwards, not unless the correlations can be used for making new predictions. Otherwise it's like betting on a race after it's been run."

"Kingsley's ideas have many very interesting neurological implications," McNeil remarked. "Communication for us is a matter of extreme difficulty. We ourselves have to make a translation of the electrical activity—essentially D.C. activity—in our brains. To do this quite a bit of the brain is given over to the control of the lip muscles and of the vocal cords. Even so our translation is very incomplete. We don't do too badly perhaps in conveying simple ideas, but the conveying of emotions is very difficult. Kingsley's little beasts could, I suppose, convey emotions too, and that's another reason why it's rather meaningless to talk of separate individuals. It's rather terrifying to realise that everything we've been talking about tonight and conveying so inadequately from one to another could be communicated with vastly greater precision and understanding among Kingsley's little beasts in about a hundredth of a second."

"I'd like to follow the idea of separate individuals a little further," said Barnett, turning to Kingsley. "Would you think of each individual in the Cloud as building a radiative transmitter of some sort?"

"Not as *building* a transmitter. Let me describe how I see biological evolution taking place within the Cloud. At an early stage I think there would be a whole lot of more or less separate disconnected individuals. Then communication would develop, not by a deliberate inorganic building of a means of radiative transmission, but through a slow biological development. The individuals would develop a means of radiative transmission as a biological organ, rather as we have developed a mouth, tongue, lips, and vocal cords. Communication would improve to a degree that we can scarcely contemplate. A thought would no sooner be thought than it would be communicated. An emotion would no sooner be experienced than it would be shared. With this would come a submergence of the individual and an evolution into a coherent whole. The beast, as I visualise it, need not be located in a particular place in the Cloud. Its different parts may be spread through the Cloud, but I regard it as a neurological unity, interlocked by a communication system in which signals are transmitted back and forth at a speed of 186,000 miles a second."

"We ought to get down to considering those signals more closely. I suppose they'd have to have a longish wave-length. Ordinary light presumably would be useless since the Cloud is opaque to it," said Leicester.

"My guess is that the signals are radio waves," went on Kingsley. "There's a good reason why it should be so. To be really efficient one must have complete phase control in a communication system. This can be done with radio waves, but not so far as we know with shorter wave-lengths."

McNeil was excited.

"Our radio transmissions!" he exclaimed. "They'd have interfered with the beast's neurological control."

"They would if they'd been allowed to."

"What d'you mean, Chris?"

"Well, the beast hasn't only to contend with our transmissions, but with the whole welter of cosmic radio waves. From all quarters of the Universe there'd be radio waves interfering with its neurological activity unless it had developed some form of protection."

"What sort of protection have you in mind?"

"Electrical discharges in the outer part of the Cloud causing sufficient ionisation to prevent the entry of external radio waves. Such a protection would be as essential as the skull is to the human brain."

Aniseed smoke was rapidly filling the room. Marlowe suddenly found his pipe too hot to hold and put it down gingerly.

"My God, you think this explains the rise of ionisation in the atmosphere, when we switch on our transmitters?"

"That's the general idea. We were talking earlier on about a feedback mechanism. That I imagine is just what the beast has got. If any external waves get in too deeply, then up go the voltages and away go the discharges until the waves can get in no farther."

"But the ionisation takes place in our own atmosphere."

"For this purpose I think we can regard our atmosphere as a part of the Cloud. We know from the shimmering of the night sky that gas extends all the way from the Earth to the denser parts of the Cloud, the disk-like parts. In short we're inside the Cloud, electronically speaking. That, I think, explains our communication troubles. At an earlier stage, when we were outside the Cloud, the beast didn't protect itself by ionising our atmosphere, but through its outer electronic shield. But once we got inside the shield the discharges began to occur in our own atmosphere. The beast has been boxing-in our transmissions."

"Very fine reasoning, Chris," said Marlowe.

"Hellish fine," nodded Alexandrov.

"How about the one centimetre transmissions? They went through all right," Weichart objected.

"Although the chain of reasoning is getting rather long there's a suggestion that one can make on that. I think it's worth making because it suggests the next action we might take. It seems to me most unlikely that this Cloud is unique. Nature doesn't work in unique examples. So let's suppose there are lots of these beasts inhabiting the Galaxy. Then I would expect communication to occur between one cloud and another. This would imply that some wavelengths would be required for external communication purposes, wave-lengths that could penetrate into the Cloud and would do no neurological harm."

"And you think one centimetre may be such a wave-length?"

"That's the general idea."

"But then why was there no reply to our one centimetre transmission?" asked Parkinson.

"Perhaps because we sent no message. There'd be no point in replying to a perfectly blank transmission."

"Then we ought to start sending pulsed messages on the one centimetre," exclaimed Leicester. "But how can we expect the Cloud to decipher them?"

"That's not an urgent problem to begin with. It will be obvious that our transmissions contain information—that will be clear from the frequent repetition of various patterns. As soon as the Cloud realises that our transmissions have intelligent control behind them I think we can expect some sort of reply. How long will it take to get started, Harry? You're not in a position to modulate the one centimetre yet, are you."

"No, but we can be in a couple of days, if we work night shifts. I had a sort of presentiment that I wasn't going to see my bed tonight. Come on, chaps, let's get started."

Leicester stood up, stretched himself, and ambled out. The meeting broke up. Kingsley took Parkinson on one side.

"Look, Parkinson," he said, "there's no need to go gabbling about this until we know more about it."

"Of course not. The Prime Minister suspects I'm off my head as it is."

"There is one thing that you might pass on, though. If London, Washington, and the rest of the political circus could get ten centimetre transmitters working, it's just possible that they might avoid the fade-out trouble."

When Kingsley and Ann Halsey were alone later that night, Ann remarked:

"How on earth did you come on such an idea, Chris?"

"Well, it's pretty obvious really. The trouble is that we're all inhibited against such thinking. The idea that the Earth is the only possible abode of life runs pretty deep in spite of all the science fiction and kid's comics. If we had been able to look at the business with an impartial eye we should have spotted it long ago. Right from the first, things have gone wrong and they've gone wrong according to a systematic sort of pattern. Once I overcame the psychological block, I saw all the difficulties could be removed by one simple and entirely plausible step. One by one the bits of the puzzle fitted into place. I think Alexandrov probably had the same idea, only his English is a bit on the terse side."

"On the bloody terse side, you mean. But seriously, do you think this communication business will work?"

"I very much hope so. It's quite crucial that it should."

"Why do you say that?"

"Think of the disasters the Earth has suffered so far, without the Cloud taking any purposive steps against us. A bit of reflection from its surface nearly roasted us. A short obscuration of the Sun nearly froze us. If the merest tiny fraction of the energy controlled by the Cloud should be directed against us we should be wiped out, every plant and animal."

"But why should that happen?"

"How can one tell? Do you think of the tiny beetle or the ant that you crush under your foot on an afternoon's walk? One of those gas bullets that hit the Moon three months ago would finish us. Sooner or later the Cloud will probably let fly with some more of 'em. Or we might be electrocuted in some monstrous discharge."

"Could the Cloud really do that?"

"Easily. The energy that it controls is simply monstrous. If we can get some sort of a message across, then perhaps the Cloud will take the trouble to avoid crushing us under its foot."

"But why should it bother?"

"Well, if a beetle were to say to you, 'Please, Miss Halsey, will you avoid treading here, otherwise I shall be crushed,' wouldn't you be willing to move your foot a trifle?"

Scientists often stress that there is no single scientific method. Bridgman emphasizes this freedom to choose between many procedures, a freedom essential to science.

3

On Scientific Method

Percy W. Bridgman

1949

IT SEEMS TO ME that there is a good deal of ballyhoo about scientific method. I venture to think that the people who talk most about it are the people who do least about it. Scientific method is what working scientists do, not what other people or even they themselves may say about it. No working scientist, when he plans an experiment in the laboratory, asks himself whether he is being properly scientific, nor is he interested in whatever method he may be using *as method.* When the scientist ventures to criticize the work of his fellow scientist, as is not uncommon, he does not base his criticism on such glittering generalities as failure to follow the "scientific method," but his criticism is specific, based on some feature characteristic of the particular situation. The working scientist is always too much concerned with getting down to brass tacks to be willing to spend his time on generalities.

Scientific method is something talked about by people standing on the outside and wondering how the scientist manages to do it. These people have been able to uncover various generalities applicable to at least most of what the scientist does, but it seems to me that these generalities are not very pro-

*From The Teaching Scientist, December 1949, written at the request of the editor.

found, and could have been anticipated by anyone who knew enough about scientists to know what is their primary objective. I think that the objectives of all scientists have this in common—that they are all trying to get the correct answer to the particular problem in hand. This may be expressed in more pretentious language as the pursuit of truth. Now if the answer to the problem is correct there must be some way of knowing and proving that it is correct—the very meaning of truth implies the possibility of checking or verification. Hence the necessity for checking his results always inheres in what the scientist does. Furthermore, this checking must be exhaustive, for the truth of a general proposition may be disproved by a single exceptional case. A long experience has shown the scientist that various things are inimical to getting the correct answer. He has found that it is not sufficient to trust the word of his neighbor, but that if he wants to be sure, he must be able to check a result for himself. Hence the scientist is the enemy of all authoritarianism. Furthermore, he finds that he often makes mistakes himself and he must learn how to guard against them. He cannot permit himself any preconception as to what sort of results he will get, nor must he allow himself to be influenced by wishful thinking or any personal bias. All these things together give that "objectivity" to science which is often thought to be the essence of the scientific method.

But to the working scientist himself all this appears obvious and trite. What appears to him as the essence of the situation is that he is not consciously following any prescribed course of action, but feels complete freedom to utilize any method or device whatever which in the particular situation before him seems likely to yield the correct answer. In his attack on his specific problem he suffers no inhibitions of precedent or authority, but is completely free to adopt any course that his ingenuity is capable of suggesting to him. No one standing on the outside can predict what the individual scientist will do or what method he will follow. In short, science is what scientists do, and there are as many scientific methods as there are individual scientists.

This is Polya's one-page summary of his book in which he discusses strategies and techniques for solving problems. Polya's examples are from mathematics, but his ideas are useful in solving physics problems also.

4

How to Solve It

George Polya

1957

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition?

Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.

Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.

Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

The advice is directed primarily to the student planning a career in the sciences, but it should be of interest to a wider group.

5

Four Pieces of Advice to Young People

Warren Weaver

1966

One of the great prerogatives of age is the right to give advice to the young. Of course, the other side of the coin is that one of the prerogatives of youth is to disregard this advice. But...I am going to give you four pieces of advice, and you may do with all four of them precisely what you see fit.

The first one is this: I urge each one of you not to decide prematurely what field of science, what specialty of science you are going to make your own. Science moves very rapidly. Five years from now or ten years from now there will be opportunities in science which are almost not discernible at the present time. And, I think there are also, of course, fads in science. Science goes all out at any one moment for work in one certain direction and the other fields are thought of as being rather old-fashioned. But, don't let that fool you. Sometimes some of these very old problems turn out to be extremely significant.

May I just remind you that there is no physical entity that the mind of man has thought about longer than the phenomenon of light. One would ordinarily say that it would be simply impossible at the present day for someone to sit down and get a brand new idea about light, because think of the thousands of scientists that have worked on that subject. And yet, you see this is what two scientists did only just a few years ago when the laser was invented. They got a brand new idea about light and it has turned out to be a phenomenally important idea.

So, I urge you not to make up your minds too narrowly, too soon. Of course, that means that what you ought to do is to be certain that you get a very solid basic foundation in science so that you can then adjust yourselves to the opportunities of the future when they arise. What is that basic foundation?

Well, of course, you don't expect me to say much more than mathematics, do you? Because I was originally trained as a mathematician and mathematics is certainly at the bottom of all this. But I also mean the fundamentals of physics and the fundamentals of chemistry. These two, incidentally, are almost indistinguishable nowadays from the fundamentals of biology.

The second piec of advice that I will just mention to you because maybe some of you are thinking too exclusively in terms of a career in research. In my judgment there is no life that is possible to be lived on this planet that is more pleasant and more rewarding than the combined activity of teaching and research.

I hope very much that many of you look forward to becoming teachers. It is a wonderful life. I don't know of any better one myself, any more pleasant one, or any more rewarding one. And the almost incredible fact is that they even pay you for it. And, nowadays, they don't pay you too badly. Of course, when I started, they did. But, nowadays, the pay is pretty good.

My third piece of advice—may I urge every single one of you to prepare yourself not only to be a scientist, but to be a scientist-citizen. You have to accept the responsibilities of citizenship in a free democracy. And those are great responsibilities and because of the role which science plays in our modern world, we need more and more people who understand science but who are also sensitive to and aware of the responsibilities of citizenship.

And the final piece of advice is—and maybe this will surprise you: Do not overestimate science, do not think that science is all that there is, do not concentrate so completely on science that you end up by living a warped sort of life. Science is not all that there is, and science is not capable of solving all of life's problems. There are also many more very important problems that science cannot solve.

And so I hope very much there's nobody in this room who is going to spend the next seven days without reading some poetry. I hope that there's nobody in this room that's going to spend the next seven days without listening to some music, some good music, some modern music, some music. I hope very much that there's nobody here who is not interested in the creative arts, interested in drama, interested in the dance. I hope that you interest yourselves seriously in religion, because if you do not open your minds and open your activities to this range of things, you are going to lead too narrow a life.

The size of an animal is related to such physical factors as gravity and temperature. For most animals there appears to be an optimum size.

6 On Being the Right Size

J. B. S. Haldane

1928

> *From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight, so also it would be impossible to build up the bony structures of men, horses, or other animals so as to hold together and perform their normal functions if these animals were to be increased enormously in height; for this increase in height can be accomplished only by employing a material which is harder and stronger than usual, or by enlarging the size of the bones, thus changing their shape until the form and appearance of the animals suggest a monstrosity. This is perhaps what our wise Poet had in mind, when he says, in describing a huge giant:*
>
> *"Impossible it is to reckon his height*
> *So beyond measure is his size."*
>
> —GALILEO GALILEI

THE most obvious differences between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form.

Let us take the most obvious of possible cases, and consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated *Pilgrim's Progress* of my childhood. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step. This was doubtless why they were sitting down in the picture I remember. But it lessens one's respect for Christian and Jack the Giant Killer.

To turn to zoology, suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners.

Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away. A rat would probably be killed, though it can fall safely from the eleventh story of a building; a man is killed, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insect, therefore, is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-long-legs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as every one knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, contrive to be unwettable, the majority keep well away from their drink by means of a long proboscis.

Of course tall land animals have other difficulties. They have to pump their blood to greater heights than a man and, therefore, require a larger blood pressure and tougher blood-vessels. A great many men die from burst arteries, especially in the brain, and this danger is presumably still greater for an elephant or a giraffe. But animals of all kinds find difficulties in size for the following reason. A typical small animal, say a microscopic worm or rotifer, has a smooth skin through which all the oxygen it requires can soak in, a straight gut with sufficient surface to absorb its food, and a simple kidney. Increase its dimensions tenfold in every direction, and its weight is increased a thousand times, so that if it is to use its muscles as efficiently as its miniature counterpart, it will need a thousand times as much food and oxygen per day and will excrete a thousand times as much of waste products.

Now if its shape is unaltered its surface will be increased only a hundredfold, and ten times as much oxygen must enter per minute through each square millimetre of skin, ten times as much food through each square millimetre of intestine. When a limit is reached to their absorptive powers their surface has to be increased by some special device. For example, a part of the skin may be drawn out into tufts to make gills or pushed in to make lungs, thus increasing the oxygen-absorbing surface in proportion to the animal's bulk. A man, for example, has a hundred square yards of lung. Similarly, the gut, instead of being smooth and straight, becomes coiled and develops a velvety surface, and other organs increase in complication. The higher animals are not larger than the lower because they are more complicated. They are more complicated because they are larger. Just the same is true of plants. The simplest plants, such as the green algae growing in stagnant water or on the bark of trees, are mere round cells. The higher plants increase their surface by putting out leaves and roots. Comparative anatomy is largely the story of the struggle to increase surface in proportion to volume.

Some of the methods of increasing the surface are useful up to a point, but not capable of a very wide adaptation. For example, while vertebrates carry the oxygen from the gills or lungs all over the body in the blood, insects take air directly to every part of their body by tiny blind tubes called tracheae which open to the surface at many different points. Now, although by their breathing movements they can renew the air in the outer part of the tracheal system, the oxygen has to penetrate the finer branches by means of diffusion. Gases can diffuse easily through very small distances, not many times larger than the average length travelled by a gas molecule between collisions with other molecules. But when such vast journeys—from the point of view of a molecule—as a quarter of an inch have to be made, the process becomes slow. So the portions of an insect's body more than a quarter of an inch from the air would always be short of oxygen. In consequence hardly any insects are much more than half an inch thick. Land crabs are built on the same general plan as insects, but are much clumsier. Yet like ourselves they carry oxygen around in their blood, and are therefore able to grow far larger than any insects. If the insects had hit on a plan for driving air through their tissues instead of letting it soak in, they might well have become as large as lobsters, though other considerations would have prevented them from becoming as large as man.

Exactly the same difficulties attach to flying. It is an elementary principle of aeronautics that the minimum speed needed to keep an aeroplane of a given shape in the air varies as the square root of its length. If its linear dimensions are increased four times, it must fly twice as fast. Now the power needed for the minimum speed increases more rapidly than the weight of the machine. So the larger aeroplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up. Applying the same principles to the birds, we find that the limit to their size is soon reached. An angel whose muscles developed no more power weight for weight than those of an eagle or a pigeon would require a breast projecting for about four feet to house the muscles engaged in working its wings, while to economize in weight, its legs would have to be reduced to mere stilts. Actually a large bird such as an eagle or kite does not keep in the air mainly by moving its wings. It is generally to be seen soaring, that is to say balanced on a rising column of air. And even soaring becomes more and more difficult with increasing size. Were this not the case eagles might be as large as tigers and as formidable to man as hostile aeroplanes.

But it is time that we passed to some of the advantages of size. One of the most obvious is that it enables one to keep warm. All warm-blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. For the same reason small animals cannot live in cold countries. In the arctic regions there are no reptiles or amphibians, and no small mammals. The smallest mammal in Spitzbergen is the fox. The small birds fly away in the winter, while the insects die, though their eggs can survive six months or more of frost. The most successful mammals are bears, seals, and walruses.

Similarly, the eye is a rather inefficient organ until it reaches a large size. The back of the human eye on which an image of the outside world is thrown, and which corresponds to the film of a camera, is composed of a mosaic of 'rods and cones' whose diameter is little more than a length of an average light wave. Each eye has about half a million, and for two objects to be distinguishable their images must fall on separate rods or cones. It is obvious that with fewer but larger rods and cones we should see less distinctly. If they were twice as broad two points would have to be twice as far apart before we could distinguish them at a given distance. But if their size were diminished and their number increased we should see no better. For it is impossible to form a definite image smaller than a wave-length of light. Hence a mouse's eye is not a small-scale model of a human eye. Its rods and cones are not much smaller than ours, and therefore there are far fewer of them. A mouse could not distinguish one human face from another six feet away. In order that they should be of any use at all the eyes of small animals have to be much larger in proportion to their bodies than our own. Large animals on the other hand only require relatively small eyes, and those of the whale and elephant are little larger than our own.

For rather more recondite reasons the same general principle holds true of the brain. If we compare the brain-weights of a set of very similar animals such as the cat, cheetah, leopard, and tiger, we find that as we quadruple the body-weight the brain-weight is only doubled. The larger animal with proportionately larger bones can economize on brain, eyes, and certain other organs.

Such are a very few of the considerations which show that for every type of animal there is an optimum size. Yet although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact the height to which an animal can jump is more nearly independent of its size than proportional to it. A flea can jump about two feet, a man about five. To jump a given height, if we neglect the resistance of the air, requires an expenditure of energy proportional to the jumper's weight. But if the jumping muscles form a constant fraction of the animal's body, the energy developed per ounce of muscle is independent of the size, provided it can be developed quickly enough in the small animal. As a matter of fact an insect's muscles, although they can contract more quickly than our own, appear to be less efficient; as otherwise a flea or grasshopper could rise six feet into the air.

And just as there is a best size for every animal, so the same is true for every human institution. In the Greek type of democracy all the citizens could listen to a series of orators and vote directly on questions of legislation. Hence their philosophers held that a small city was the largest possible democratic state. The English invention of representative government made a democratic nation possible, and the possibility was first realized in the United States, and later elsewhere. With the development of broadcasting it has once more become possible for every citizen to listen to the political views of representative orators, and the future may perhaps see the return of the national state to the Greek form of democracy. Even the referendum has been made possible only by the institution of daily newspapers.

To the biologist the problem of socialism appears largely as a problem of size. The extreme socialists desire to run every nation as a single business concern. I do not suppose that Henry Ford would find much difficulty in running Andorra or Luxembourg on a socialistic basis. He has already more men on his pay-roll than their population. It is conceivable that a syndicate of Fords, if we could find them, would make Belgium Ltd. or Denmark Inc. pay their way. But while nationalization of certain industries is an obvious possibility in the largest of states, I find it no easier to picture a completely socialized British Empire or United States than an elephant turning somersaults or a hippopotamus jumping a hedge.

Not only the scientist is interested in motion. This article comments briefly on references to motion in poetry.

7 Motion in Words

James B. Gerhart and Rudi H. Nussbaum

1966

Man began describing movement with words long before there were physicists to reduce motion to laws. Our age-old fascination with moving things is attested to by the astonishing number of words we have for motion. We have all kinds of words for all kinds of movement: special words for going up, others for coming down; words for fast motion, words for slow motion. A thing going up may rise, ascend, climb, or spring. Going down again, it may fall or descend; sink, subside, or settle; dive or drop; plunge or plop; topple, totter, or merely droop. It may twirl, whirl, turn and circle; rotate, gyrate; twist or spin; roll, revolve and wheel. It may oscillate, vibrate, tremble and shake; tumble or toss, pitch or sway; flutter, jiggle, quiver, quake; or lurch, or wobble, or even palpitate. All these words tell some motion, yet every one has its own character. Some of them you use over and over in a single day. Others you may merely recognize. And still they are but a few of our words for motion. There are special words for the motions of particular things. Horses, for example, trot and gallop and canter while men run, or stride, or saunter. Babies crawl and creep. Tides ebb and flow, balls bounce, armies march. Other words tell the quality of motion, words like swift or fleet, like calm and slow.

Writers draw vivid mental pictures for the reader with words alone. Here is a poet's description of air flowing across a field on a hot day:

> There came a wind like a bugle:
> It quivered through the grass,
> and a green chill upon the heat
> so ominous did pass.
>
> Emily Dickinson

Or again, the motion of the sea caused by the gravitational attraction of the moon:

> The western tide crept up along
> the sand,
> and o'er and o'er the sand,
> and round and round the sand,
> as far as the eye could see.
>
> Charles Kingsley,
> The Sands of Dee

Or, swans starting into flight:

> I saw . . . all suddenly mount
> and scatter wheeling in great
> broken rings
> upon their clamorous wings.
>
> W. B. Yeats,
> The Wild Swans at Coole

Sometimes just a single sentence will convey the whole idea of motion:

> Lightly stepped a yellow star
> to its lofty place
>
> Emily Dickinson

Or, this description of a ship sailing:

> She walks the water like a thing
> of life
>
> Byron, The Corsair

How is it that these poets describe motion? They recall to us what we have seen; they compare different things through simile and metaphor; they rely on the reader to share their own emotions, and they invite him to recreate an image of motion in his own mind. The poet has his own precision which is not the scientist's precision. Emily Dickinson well knew it was the grass, not the wind, that quivered, and that stars don't step. Byron never saw a walking boat. But this is irrelevant. All of us can appreciate and enjoy their rich images and see that they are true to the nature of man's perception, if not to the nature of motion itself.

From time to time a physicist reading poetry will find a poem which describes something that he has learned to be of significance to his, the physicist's description. Here is an example:

> A ball will bounce, but less and
> less. It's not a light-hearted
> thing, resents its own resilience.
> Falling is what it loves, . . .
>
> Richard Wilbur, Juggler

Relativity is implicit in this next example:

> The earth revolves with me, yet
> makes no motion.
> The stars pale silently in a coral
> sky.
> In a whistling void I stand before
> my mirror unconcerned, and tie
> my tie.
>
> Conrad Aiken,
> Morning Song of Senlin

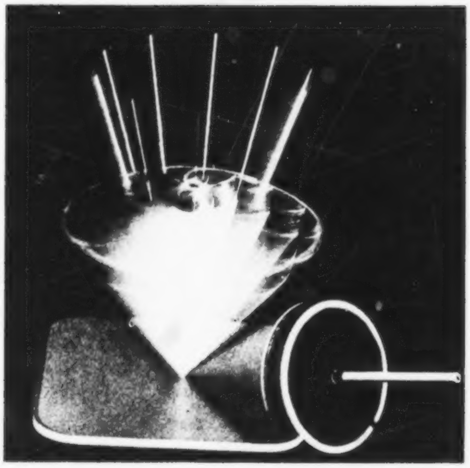


Fig. 1.10 Multiple-flash photograph showing the precession of a top.

The poet's description of motion is a rich, whole vision, filled with both his perceptions and his responses. Yet complete as it is, the poetic description is far from the scientific one. Indeed, when we compare them, it is easy to forget they deal with the same things. Just how does the scientific view of motion differ? And to what purpose? Let's try to answer these questions by shifting slowly from the poet's description to the scientist's. As a first step, read this excerpt from a biography of a physicist of the last century, Lord Kelvin. The biographer is trying to convey the electric quality of Kelvin's lectures to his University classes. He describes a lecture on tops (referred to as gyrostats here):

> The vivacity and enthusiasm of the Professor at that time was very great. The animation of his countenance as he looked at a gyrostat spinning, standing on a knife edge on a glass plate in front of him, and leaning over so that its center of gravity was on one side of the point of support; the delight with which he showed that hurrying of the precessional motion caused the gyrostat to rise, and retarding the precessional motion caused the gyrostat to fall, so that the freedom to precess was the secret of its not falling; the immediate application of the study of the gyrostat to the explanation of the precession of the equinoxes, and illustration by a model . . . - all these delighted his hearers, and made the lecture memorable.
>
> Andrew Gray, Lord Kelvin, An Account of his Scientific Life and Work

This paragraph by Gray deals with motion, but still it is more concerned with human responses - Kelvin's obvious pleasure in watching the top, and his student's evident delight in watching both Kelvin and Kelvin's top. At the same time it says much about the top's movement, hints at the reasons behind it, and mentions how understanding the top has led to understanding the precession of the earth's axis in space.

Gray used some of the everyday words for motion: rise, fall, spin, hurry, retard. But he used other words and other phrases, too - more technical, less familiar: precess, center of gravity, equinoxes. Technical words are important for a scientific description of motion. When the scientist has dissected a motion and laid out its components, the need for new terms enters, the need for words with more precise meanings, words not clothed with connotations of emotional response. Still, the scientist never can (and never really wants to), lose the connotations of common words entirely. For example, here is Lord Kelvin's attempt to define precession (see Fig. 1.10), in the sense that Gray used it:

> This we call positive precessional rotation. It is the case of a common spinning-top (peery), spinning on a very fine point which remains at rest in a hollow or hole bored

> by itself; not sleeping upright, nor nodding, but sweeping its axis round in a circular cone whose axis is vertical.
>
> William Thomson (Lord Kelvin) and P. G. Tait, Treatise on Natural Philosophy

This definition is interesting in several ways. For one thing, it seems strange today that Kelvin, a Scot, should feel the need to explain "spinning-top" by adding "peery," an obscure word to most of us, but one that Kelvin evidently thought more colloquial. Think for a moment of how Kelvin went about his definition. He used the words of boys spinning tops for fun, who then, and still today, say a top sleeps when its axis is nearly straight up, and that it nods as it slows and finally falls. He reminded his readers of something they all had seen and of the everyday words for it. (He obviously assumed that most of his readers once played with tops.) In fact, this is the best way to define new words - to remind the reader of something he knows already and with words he might use himself. Having once given this definition Kelvin never returns to the picture he employed. It is clear, though, that when he wrote, "positive precessional rotation," he brought this image to his own mind, and that he expected his readers to do the same.

Of course, it is not necessary to use as many words as Kelvin did to define precession. Another, more austere, and to some, more scientific definition is this:

> When the axis of the top travels round the vertical making a constant angle i with it, the motion is called steady or precessional.
>
> E. J. Routh, Treatise on the Dynamics of a System of Rigid Bodies

All that refers to direct, human experience is missing here. The top is now just something with an axis, no longer a bright-painted toy spinning on the ground. And it is not the top that moves, but its axis, an imagined line in space, and this line moves about another imagined line, the vertical. There is no poetry here, only geometry. This is an exact, precise, and economical definition, but it is abstract, and incomplete. It does not describe what anyone watching a real top sees. In fact, it is only a few abstractions from the real top's motion on which the physicist-definer has concentrated his attention.

The twentieth century artist has been able to exploit his interest in motion in various ways in works of art.

8

Representation of Movement

Gyorgy Kepes

1944

Matter, the physical basis of all spatial experience and thus the source material of representation, is kinetic in its very essence. From atomic happenings to cosmic actions, all elements in nature are in perpetual interaction—in a flux complete. We are living a mobile existence. The earth is rotating; the sun is moving; trees are growing; flowers are opening and closing; clouds are merging, dissolving, coming and going; light and shadow are hunting each other in an indefatigable play; forms are appearing and disappearing; and man, who is experiencing all this, is himself subject to all kinetic change. The perception of physical reality cannot escape the quality of movement. The very understanding of spatial facts, the meaning of extension or distances, involves the notion of time—a fusion of space-time which is movement. "Nobody has ever noticed a place except at a time or a time except at a place," said Minkowsky in his *Principles of Relativity.*

The sources of movement perception

As in a wild jungle one cuts new paths in order to progress further, man builds roads of perception on which he is able to approach the mobile world, to discover order in its relationships. To build these avenues of perceptual grasp he relies on certain natural factors. One is the nature of the retina, the sensitive surface on which the mobile panorama is projected. The second is the sense of movement of his body—the kinesthetic sensations of his eye muscles, limbs, head, which have a direct correspondence with the happenings around him. The third is the memory association of past experience, visual and non-visual; his knowledge about the laws of the physical nature of the surrounding object-world.

The shift of the retinal image

We perceive any successive stimulation of the retinal receptors as movement, because such progressive stimulations are in dynamic interaction with fixed stimulations, and therefore the two different types of stimulation can be perceived in a unified whole only as a dynamic process, movement. If the retina is stimulated with stationary impacts that follow one another in rapid succession, the same sensation of optical movement is induced. Advertising displays with their rapidly flashing electric bulbs are perceived in continuity through the persistence of vision and therefore produce the sensation of movement, although the spatial position of the light bulbs is stationary. The movement in the motion picture is based upon the same source of the visual perception.

The changes of any optical data indicating spatial relationships, such as size, shape, direction, interval, brightness, clearness, color, imply motion. If the retinal image of any of these signs undergoes continuous regular change, expansion or contraction, progression or graduation, one perceives an approaching or receding, expanding or contracting movement. If one sees a growing or disappearing distance between these signs, he perceives a horizontal or vertical movement.

"Suppose for instance, that a person is standing still in a thick woods, where it is impossible for him to distinguish, except vaguely and roughly in a mass of foliage and branches all around him, what belongs to one tree and what to another, and how far the trees are separated. The moment he begins to move forward, however, everything disentangles itself and immediately he gets an apperception of the content of the woods and the relationships of objects to each other in space."*

From a moving train, the closer the object the faster it seems to move. A far-away object moves slowly and one very remote appears to be stationary. The same phenomenon, with a lower relative velocity, may be noticed in walking, and with a still higher velocity in a landing aeroplane or in a moving elevator.

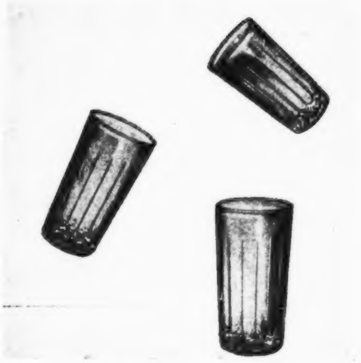
The role of relative velocity

The velocity of motion has an important conditioning effect. Motion can be too fast or too slow to be perceived as such by our limited sensory receiving set. The growth of trees or of man, the opening of flowers, the evaporation of water are movements beyond the threshold of ordinary visual grasp. One does not see the movement of the hand of a watch. of a ship on a distant horizon. An aeroplane in the highest sky seems to hang motionless. No one can see the traveling of light as such. In certain less rapid motions beyond the visual grasp, one is able, however, to observe the optical transformation of movement into the illusion of a solid. A rapidly whirled torch loses its characteristic physical extension, but it submerges into another three-dimensional-appearing solid—into the virtual volume of a cone or a sphere. Our inability to distinguish sharply beyond a certain interval of optical impacts makes the visual impressions a blur which serves as a bridge to a new optical form. The degree of velocity of its movement will determine the apparent density of that new form. The optical density of the visible world is in a great degree conditioned by our visual ability, which has its particular limitations.

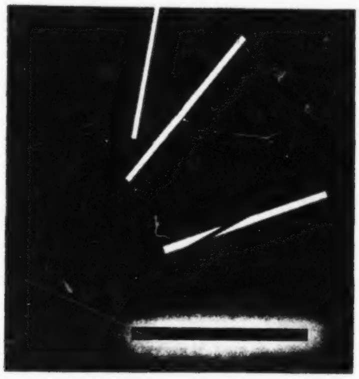
The kinesthetic sensation

When a moving object comes into the visual field, one pursues it by a corresponding movement of his eyes, keeping it in a stationary or nearly stationary position on the retina. Retinal stimulation, then, cannot alone account for the sensation of movement. Movement-experience, which is undeniably present in such a case, is induced by the sensation of muscle movements. Each individual muscle-fibre contains a nerve end, which registers every movement the muscle makes. That we are able to sense space in the dark, evaluate direction-distances in the absence of contacted bodies, is due to this muscular sensation—the kinesthetic sensation.

• *Helmholtz, Physiological Optics*



E. G. Lukacs. *Action*
From Herbert Bayer Design Class



H. L. Carpenter. *Movement* •

• *Work done for the author's course in Visual Fundamentals.*

Paul Rand. *Cover Design*

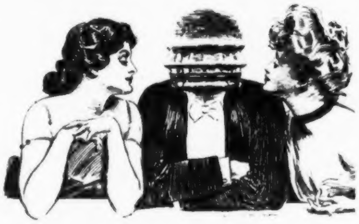
Memory sources

Experience teaches man to distinguish things and to evaluate their physical properties. He knows that bodies have weight: unsupported they will of necessity fall. When, therefore, he sees in midair a body he knows to be heavy, he automatically associates the direction and velocity of its downward course. One is also accustomed to seeing small objects as more mobile than large ones. A man is more mobile than a mountain; a bird is more frequently in motion than a tree, the sky, or other visible units in its background. Everything that one experiences is perceived in a polar unity in which one pole is accepted as a stationary background and the other as a mobile, changing figure.

Through all history painters have tried to suggest movement on the stationary picture surface, to translate some of the optical signs of movement-experience into terms of the picture-image. Their efforts, however, have been isolated attempts in which one or the other sources of movement-experience were drawn upon: the shift of the retinal image, the kinesthetic experience, or the memory of past experiences were suggested in two-dimensional terms.

These attempts were conditioned mainly by the habit of using things as the basic measuring unit for every event in nature. The constant characteristics of the things and objects, first of all the human body, animals, sun, moon, clouds, or trees, were used as the first fixed points of reference in seeking relationships in the optical turmoil of happenings.

Therefore, painters tried first to represent motion by suggesting the visible modifications of objects in movement. They knew the visual characteristics of stationary objects and therefore every observable change served to suggest movement. The prehistoric artist knew his animals, knew, for example, how many legs they had. But when he saw an animal in really speedy movement, he could not escape seeing the visual modification of the known spatial characteristics. The painter of the Altamiro caves who pictures a running reindeer with numerous legs, or the twentieth century cartoonist picturing a moving face with many superimposed profiles, is stating a relationship between what he knows and what he sees.

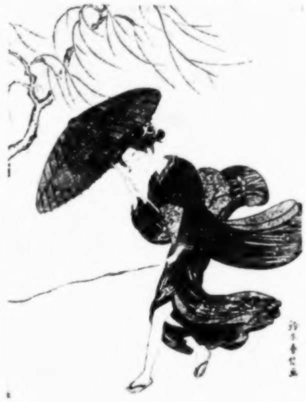


Ch. D. Gibson.
The Gentleman's Dilemma cc. 1900

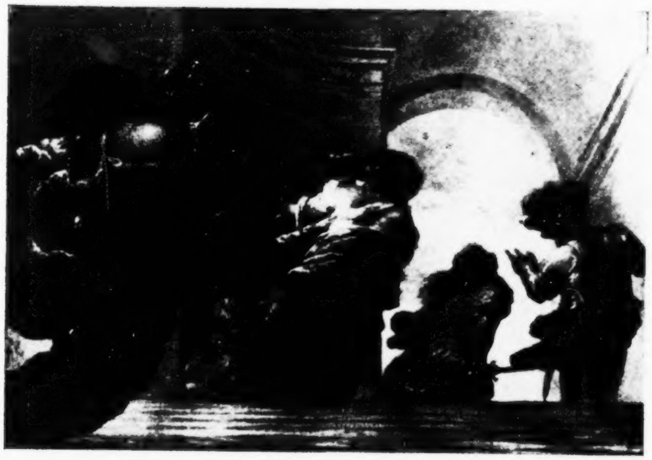
Other painters, seeking to indicate movement, utilized the expressive distortion of the moving bodies. Michaelangelo, Goya, and also Tintoretto, by elongating and stretching the figure, showed distortion of the face under the expression of strains of action and mobilized numerous other psychological references to suggest action.

The smallest movement is more possessive of the attention than the greatest wealth of relatively stationary objects. Painters of many different periods observed this well and explored it creatively. The optical vitality of the moving units they emphasized by dynamic outlines, by a vehement interplay of vigorous contrast of light and dark, and by extreme contrast of colors. In various paintings of Tintoretto, Maffei, Veronese, and Goya, the optical wealth and intensity of the moving figures are juxtaposed against the submissive, neutral, visual pattern of the stationary background.

Representation of Movement



Harunobu. ***Windy Day Under Willow***
Courtesy of The Art Institute of Chicago



Maffei. ***Painting***

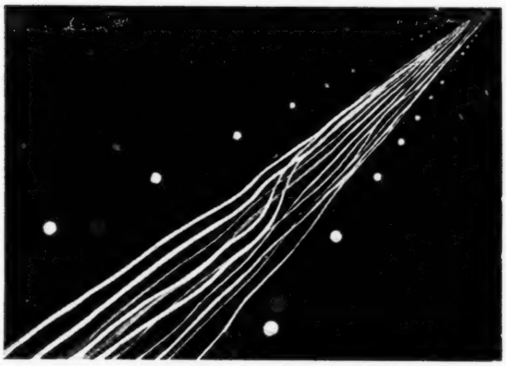
The creative exploitation of the successive stimulations of the retinal receptors in terms of the picture surface was another device many painters found useful. Linear continuance arrests the attention and forces the eye into a pursuit movement. The eye, following the line, acts as if it were on the path of a moving thing and attributes to the line the quality of movement. When the Greek sculptors organized the drapery of their figures which they represented in motion, the lines were conceived as optical forces making the eye pursue their direction.

We know that a heavy object in a background that does not offer substantial resistance will fall. Seeing such an object we interpret it as action. We make a kind of psychological qualification. Every object seen and interpreted in a frame of reference of gravitation is endowed with potential action and could appear as falling, rolling, moving. Because we customarily assume an identity between the horizontal and vertical directions on the picture surface and the main directions of space as we perceive them in our everyday experiences, every placing of an object representation on the picture surface which contradicts the center of gravity, the main direction of space—the horizontal or vertical axis—causes that object to appear to be in action. Top and bottom of the picture surface have a significance in this respect.

G. McVicker. ***Study of Linear Movement***

*Work done for the author's course
in Visual Fundamentals*

*Sponsored by The Art Director's Club
of Chicago. 1938*



Lee King. ***Study of Movement Representation***

*Work done for the author's course
in Visual Fundamentals*

School of Design in Chicago

Representation of Movement

Whereas the visual representation of depth had found various complete systems, such as linear perspective, modelling by shading, a parallel development had never taken place in the visual representation of motion. Possibly this has been because the tempo of life was comparatively slow; therefore, the ordering and representation of events could be compressed without serious repercussions in static formulations. Events were measured by things, static forms identical with themselves, in a perpetual fixity. But this static point of view lost all semblance of validity when daily experiences bombarded man with a velocity of visual impacts in which the fixity of the things, their self-identity, seemed to melt away. The more complex life became, the more dynamic relationships confronted man, in general and in particular, as visual experiences, the more necessary it became to revaluate the old relative conceptions about the fixity of things and to look for a new way of seeing that could interpret man's surroundings in their change. It was no accident that our age made the first serious search for a reformulation of the events in nature into dynamic terms. This reformulation of our ideas about the world included almost all the aspects one perceives. The interpretation of the objective world in the terms of physics, the understanding of the living organism, the reading of the inner movement of social processes, and the visual interpretation of events were, and still are, struggling for a new gauge elastic enough to expand and contract in following the dynamic changes of events.

The influence of the technological conditions

The environment of the man living today has a complexity which cannot be compared with any environment of any previous age. The skyscrapers, the street with its kaleidoscopic vibration of colors, the window-displays with their multiple mirroring images, the street cars and motor cars, produce a dynamic simultaneity of visual impression which cannot be perceived in the terms of inherited visual habits. In this optical turmoil the fixed objects appear utterly insufficient as the measuring tape of the events. The artificial light, the flashing of electric bulbs, and the mobile game of the many new types of light-sources bombard man with kinetic color sensations having a keyboard never before experienced. Man, the spectator, is himself more mobile than ever before. He rides in streetcars, motorcars and aeroplanes and his own motion gives to optical impacts a tempo far beyond the threshold of a clear object-perception. The machine man operates adds its own demand for a new way of seeing. The complicated interactions of its mechanical parts cannot be conceived in a static way; they must be perceived by understanding of their movements. The motion picture, television, and, in a great degree, the radio, require a new thinking, i.e., seeing, that takes into account qualities of change, interpenetration and simultaneity.

Man can face with success this intricate pattern of the optical events only as he can develop a speed in his perception to match the speed of his environment. He can act with confidence only as he learns to orient himself in the new mobile landscape. He needs to be quicker than the event he intends to master. The origin of the word "speed" has a revealing meaning. In original form in most languages, speed is intimately connected with success. Space and speed are, moreover, in some early forms of languages, interchangeable in meaning. Orientation, which is the basis of survival, is guaranteed by the speed of grasping the relationships of the events with which man is confronted.

Social and psychological motivations

Significantly, the contemporary attempts to represent movement were made in the countries where the vitality of living was most handicapped by outworn social conditions. In Italy, technological advances and their economical-social consequences, were tied with the relics of past ideas, institutions. The advocates of change could see no clear, positive direction. Change as they conceived it meant expansion, imperialist power policy. The advance guard of the expanding imperialism identified the past with the monuments of the past, and with the keepers of these monuments; and they tried to break, with an uninhibited vandalism, everything which seemed to them to fetter the progress toward their goals. "We want to free our country from the fetid gangrene of professors, archaeologists, guides and antique shops," proclaimed the futurist manifesto of 1909. The violence of imperialist expansion was identified with vitality; with the flux of life itself. Everything which stood in the way of this desire of the beast to reach his prey was to be destroyed. Movement, speed, velocity became their idols. Destructive mechanical implements, the armoured train, machine gun, a blasting bomb, the aeroplane, the motor car, boxing, were adored symbols of the new virility they sought.

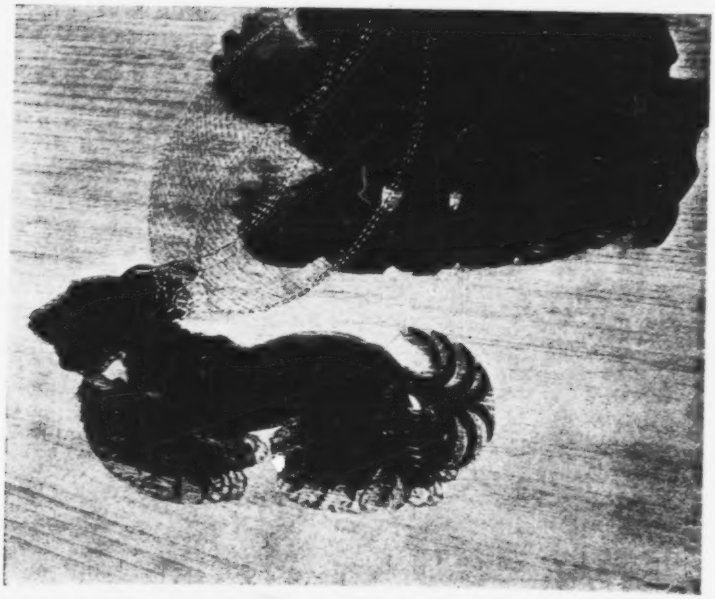
In Russia, where the present was also tied to the past and the people were struggling for the fresh air of action, interest also focused on the dynamic qualities of experience. The basic motivation of reorientation toward a kinetic expression there was quite similar to that of the Italian futurists. It was utter disgust with a present held captive by the past. Russia's painters, writers, like Russia's masses, longed to escape into a future free from the ties of outworn institutions and habits. Museums, grammar, authority, were conceived of as enemies; force, moving masses, moving machines were friends. But this revolt against stagnant traditions, this savage ridiculing of all outworn forms, opened the way for the building of a broader world. The old language, which as Mayakovsky said "was too feeble to catch up with life," was reorganized into kinetic idioms of revolutionary propaganda. The visual language of the past, from whose masters Mayakovsky asked with just scorn, "Painters will you try to capture speedy cavalry with the tiny net of contours?" was infused with new living blood of motion picture vision.

In their search to find an optical projection which conformed to the dynamic reality as they sensed and comprehended it, painters unconsciously repeated the path traced by advancing physical science.

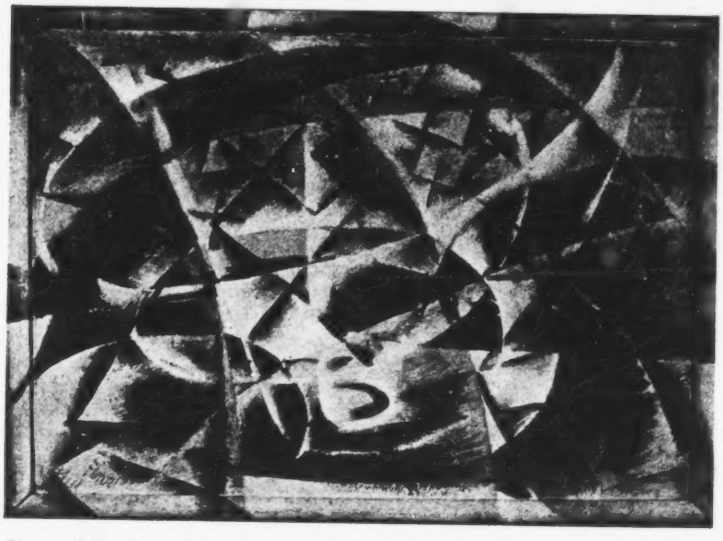
Their first step was to represent on the same picture-plane a sequence of positions of a moving body. This was basically nothing but a cataloging of stationary spatial locations. The idea corresponded to the concept of classical physics, which describes objects existing in three-dimensional space and changing locations in sequence of absolute time. The concept of the object was kept. The sequence of events frozen on the picture-plane only amplified the contradiction between the dynamic reality and the fixity of the three-dimensional object-concept.

Their second step was to fuse the different positions of the object by filling out the pathway of their movement. Objects were no longer considered as isolated, fixed units. Potential and kinetic energies were included as optical characteristics. The object was regarded to be either in active motion, indicating its direction by "lines of force." or in potential motion, pregnant with lines of force, which pointed th[illegible]ction in which the object would go if freed. The painters thus so[illegible]o picture the mechanical point of view of nature, devising optical equivalents for mass, force, and gravitation. This innovation signified important progress, because the indicated lines of forces could function as the plastic forces of two-dimensional picture-plane.

The third step was guided by desire to integrate the increasingly complicated maze of movement-directions. The chaotic jumble of centrifugal line of forces needed to be unified. Simultaneous representation of the numerous visible aspects composing an event was the new representational technique here introduced. The cubist space analysis was synchronized with the line of forces. The body of the moving object, the path of its movement and its background were portrayed in the same picture by fusing all these elements in a kinetic pattern. The romantic language of the futurist manifestos describes the method thus: "The simultaneosity of soul in a work of art; such is the exciting aim of our art. In painting a figure on a balcony, seen from within doors, we shall not confine the view to what can be seen through the frame of the window; we shall give the sum total of the visual sensation of the street, the double row of houses extending right and left the flowered balconies, etc. . . . in other words, a simultaneity of environment and therefore a dismemberment and dislocation of objects, a scattering and confusion of details independent of one and another and without reference to accepted logic," said Marinetti. This concept shows a great similarity to the idea expressed by Einstein, expounding as a physicist the space-time interpretation of the general theory of relativity. "The world of events can be described by a static picture thrown onto the background of the four dimensional time-space continuum. In the past science described motion as happenings in time, general theory of relativity interprets events existing in space-time."



Giacomo Balla. ***Dog on Leash 1912.*** *Courtesy of The Museum of Modern Art*



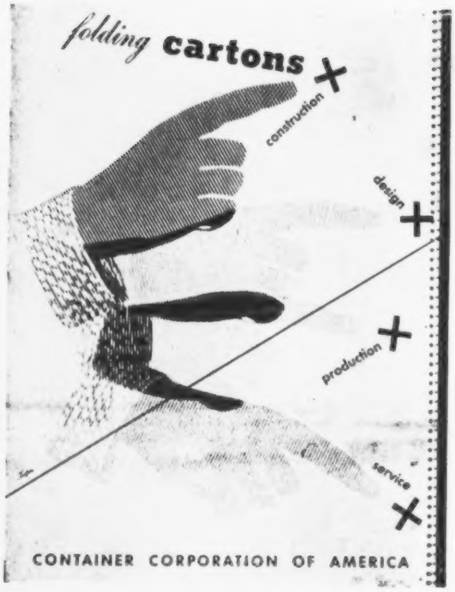
Giacomo Balla. ***Automobile and Noise.*** *Courtesy of Art of This Century*

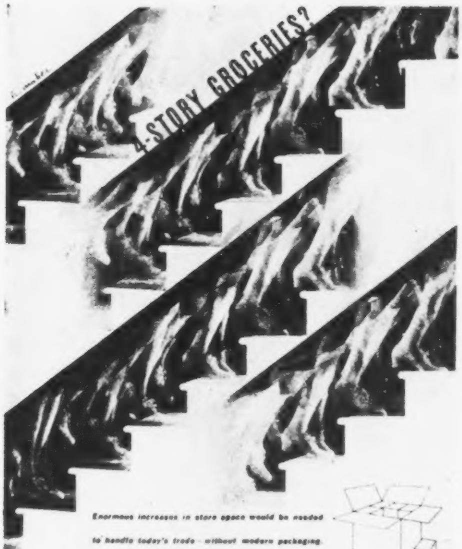
Representation of Movement

Marcel Duchamp. ***Sad Young Man in a Train.***
Courtesy of Art of This Century

Marcel Duchamp.
Nude Descending the Stairs 1912
Reproduction Courtesy
The Art Institute of Chicago

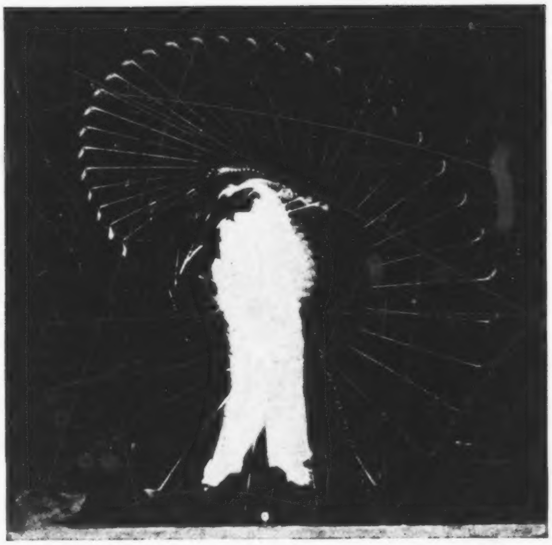
Gyorgy Kepes. ***Advertising Design 1938***
Courtesy of Container Corporation of America




4-STORY GROCERIES?
Enormous increases in store space would be needed to handle today's trade - without modern packaging.
CONTAINER CORPORATION OF AMERICA

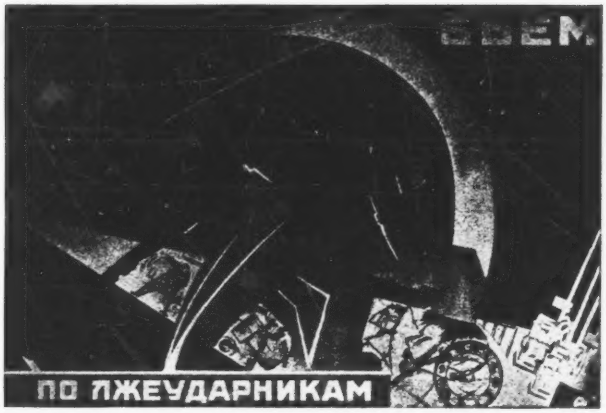
Herbert Matter. ***Advertising Design***
Courtesy of Container Corporation of America

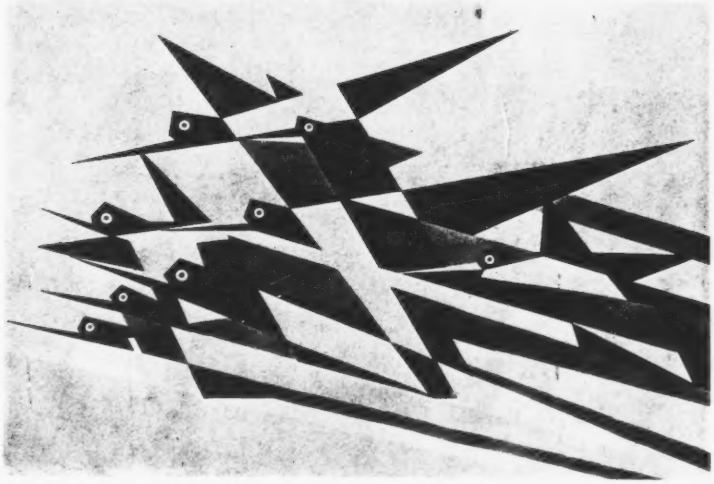
Representation of Movement



Harold E. Edgerton. *Golfer*

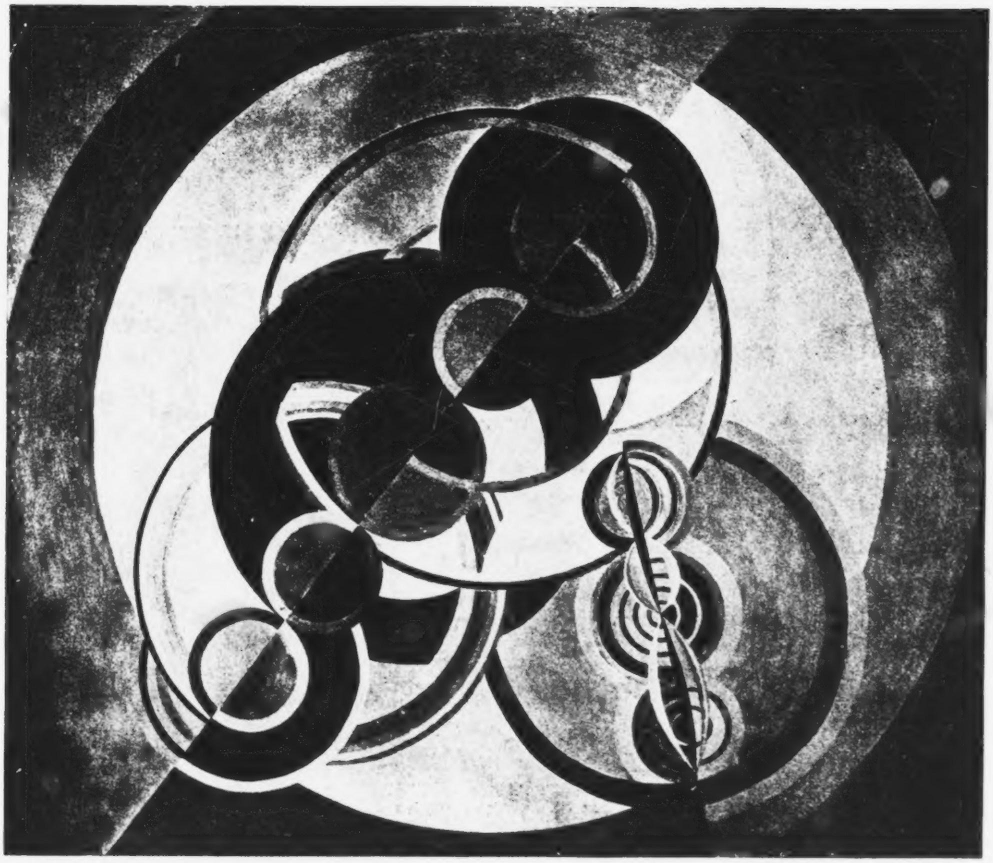
Soviet Poster





E. McKnight Kauffer. ***The Early Bird* 1919**
Courtesy of The Museum of Modern Art

Representation of Movement



Delauney. ***Circular Rhythm*** *Courtesy of The Guggenheim Museum of Non-Objective Art*

The closest approximation to representation of motion in the genuine terms of the picture-plane was achieved by the utilization of color planes as the organizing factor. The origin of color is light, and colors on the picture surface have an intrinsic tendency to return to their origin. Motion, therefore, is inherent in color. Painters intent on realizing the full motion potentialities of color believed that the image becomes a form only in the progressive interrelationships of opposing colors. Adjacent color-surfaces exhibit contrast effects. They reinforce each other in hue, saturation, and intensity.

The greater the intensity of the color-surfaces achieved by a carefully organized use of simultaneous and successive contrast, the greater their spatial movement color in regard to picture-plane. Their advancing, receding, contracting and circulating movement on the surface creates a rich variety, circular, spiral, pendular, etc., in the process of moulding them into one form which is light or, in practical terms, grey. "Form is movement," declared Delaunay. The classical continuous outline of the objects was therefore eliminated and a rhythmic discontinuity created by grouping colors in the greatest possible contrast. The picture-plane, divided into a number of contrasting color-surfaces of different hue, saturation, and intensity, could be perceived only as a form, as a unified whole in the dynamic sequence of visual perception. The animation of the image they achieved is based upon the progressive steps in bringing opposing colors into balance.

The centrifugal and centripetal forces of the contrasting color-planes move forward and backward, up and down, left and right, compelling the spectator to a kinetic participation as he follows the intrinsic spatial-direction of colors. The dynamic quality is based upon the genuine movement of plastic forces in their tendency toward balance. Like a spinning top or the running wheel of a bicycle, which can find its balance only in movement, the plastic image achieves unity in movement, in perpetual relations of contrasting colors.

A. M. Cassandre. ***Poster***

In these two chapters the elements of calculus are introduced and used to define the concept of speed. This article should be useful preparation for reading Feynman's article on Newton's Laws of Dynamics in this reader.

9 Speed

W. W. Sawyer

1961

We are going to investigate speed, the speed of a moving object. How can we see clearly what a moving object is doing? We might make a "movie" of an object moving along a straight line. Suppose we have a camera that makes a picture every tenth of a second. Suppose successive pictures are as shown in Fig. 1. What is the little object doing? Every tenth of a second, it moves up 1 inch. It seems to be moving with a steady speed of 10 inches a second.

On another occasion, we might obtain the pictures shown in Fig. 2. Here, the object advances 2 inches between each picture and the next. It has a steady speed of 20 inches a second.

Let us look at something with a varying speed. Suppose an object is accelerating. Between the first and second pictures it might cover 1

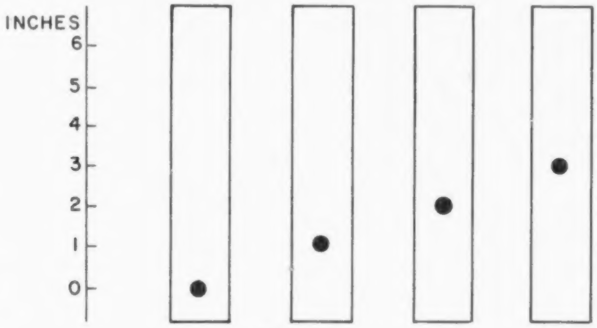


Figure 1

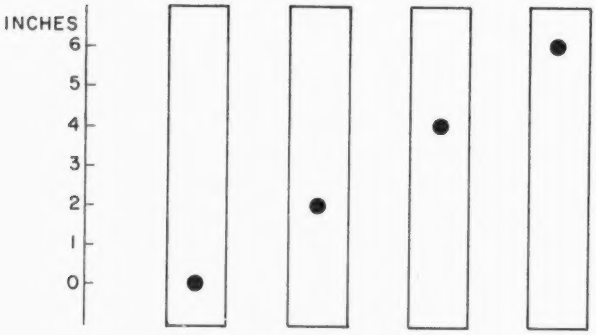


Figure 2

inch; between the second and third, 2 inches; between the third and fourth pictures, 3 inches. Its record would be as shown in Fig. 3.

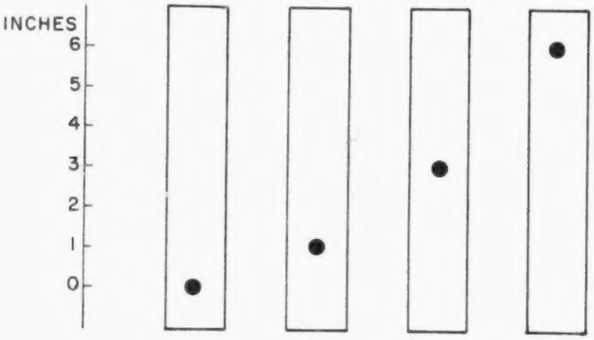


Figure 3

Already we notice certain things. (1) With steady speeds the dots lie on a straight line, (2) with accelerated motion, the dots lie on a curve.

Question 1. Figures 1 and 2 both represent objects moving with steady speeds. How could one tell, by examining these pictures, which object was moving faster? It is not necessary to bring numbers into the answer. It is possible to tell, at a single glance, which object is the faster. How?†

We can also make an object record its own motion. In Fig. 4, the object moves up and down the line *PQ*. Paper passes underneath from right to left at a steady speed; the object is inked so that it leaves a trail on the paper. If the object has a steady speed, its trail will be a straight line.

† Answers to problems will be found at the back of the book.

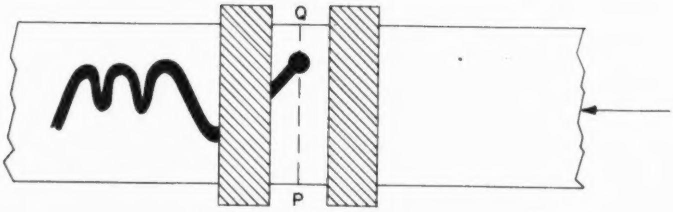


Figure 4

Question 2. Fit the records shown in Fig. 5 to the descriptions:

(a) Moving up rapidly.
(b) Moving up slowly.
(c) Stationary.
(d) Moving down slowly.
(e) Moving down fast.

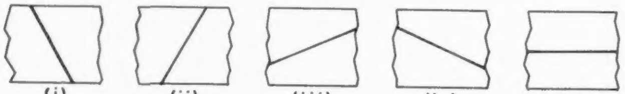

(i) (ii) (iii) (iv) (v)

Figure 5

Question 3. Fit the records shown in Fig. 6 to the descriptions:

(f) Starting from rest and gradually gaining speed upwards.
(g) Rising fast at first and gradually slowing down to rest.
(h) Starting from rest and gradually acquiring speed downwards.
(i) Falling fast at first and gradually being brought to rest.

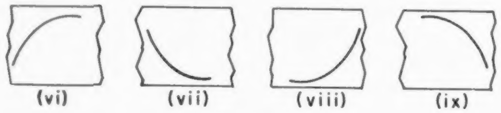

(vi) (vii) (viii) (ix)

Figure 6

No special equipment is needed, if you want to demonstrate the connection between curves and movement. The simplest thing is to draw the curve first, and then pass it behind a narrow slit; the arrangement is similar to that of Fig. 4. You will only be able to see a small part of the curve through the slit, and this will give you the impression of a point rising and falling.

This has an engineering application. If we want to make an object behave in a particular way, we can do so by means of a suitably shaped cam.

In Fig. 7, for example, the cam moves to the left at a steady pace. The rod *AB* remains at rest, until the point *C* reaches *B*. It will then begin to gather speed upward until *D* reaches *B*. When in contact with the straight part *DE*, the rod will move upwards with steady speed. The rod loses speed when in contact with the curve *EF*. Finally, it again is at rest when the section *FG* reaches it.

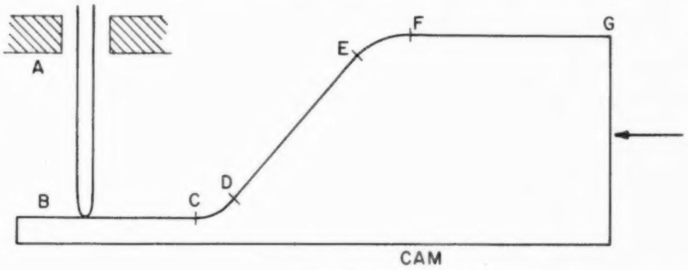


Figure 7

Curves like those in Figs. 5, 6, and 7 help us to think about movement. We can see the curves; details appear in the curves that might not be apparent in the actual movement; the curves give us something definite to look at and think about.

The work we have done also tells us something about the scope of calculus. Calculus begins as the study of speed. But in thinking about speed, we have been led to the curves drawn above. These curves could be described in terms of speed. For example, curve (viii) could be described as the curve that records the movement, when an object moves upward faster and faster. So calculus can be used not only to describe *movement* but also to describe the *shapes of curves*. Calculus was in fact so used in its earliest days. Kepler, in 1609–1619, discovered the paths in which the Earth and planets move around the Sun, and the way in which their speeds varied as they went round. Isaac Newton, in the years 1665–1687, was able to show that this was what the planets ought to do, if the sun attracted them according to the inverse-square law. Thus, with the help of calculus, he accounted for both the speeds and the curves. It impressed men very much that the complicated behavior of the solar system could be deduced from three or four very simple assumptions—Newton's laws of motion and his law of gravity. Newton's laws, and his application of calculus to astronomy, have a renewed interest today, when not only can we look at the planet Mars but some of us may be able actually to go there. Calculus would be used to calculate the possible orbits from the Earth to Mars, and to decide which orbit would require the least fuel.

Calculating Velocity

Now let us turn to some simple calculation. How do we work out the velocity of an object? Suppose, for example, a car is traveling along a straight road, a turnpike say. At 2 o'clock the mileage recorder shows 70 miles. At 5 o'clock, the mileage is 220 miles. *Suppose the car has been traveling all the time at a steady speed* (this is most unlikely in practice!). How fast has it been going? This is not a difficult question. Subtracting 70 from 220, we see that the car has gone 150 miles. Subtracting 2 from 5, we see that it has taken 3 hours to do this. We divide 150 by 3 and get 50. So the speed is 50 mph.

Our reason for doing this simple piece of arithmetic is to study the *method*, rather than the answer. We want to extract from it a formula for velocity. We bring some symbols in. Let s miles be the reading of the mileage recorder at the time t hours. Thus, $t = 2$ would indicate that the time was 2 o'clock, and $s = 70$ would indicate that the car had gone a total distance of 70 miles. The information we had in the question above could be put in a table like this:

t	2	5
s	70	220

But we want to get away from the particular numbers 2, 5, 70, 220. We want a formula for giving the velocity between any two times and any two places. So we bring in some more symbols.

> *Generalized problem.* "At a hours, the mileage is p miles. At b hours, the mileage is q miles. The car moves at a steady speed. Find its velocity, v miles an hour."

We do the same steps as we did in the particular arithmetical problem, but we replace the particular numbers by the corresponding symbols. a should appear now, where 2 appeared in the arithmetic; b replaces 5, p replaces 70, q replaces 220. The table is:

t	a	b
s	p	q

In the arithmetic, we began by subtracting 70 from 220. In the algebra, we subtract p from q. So the car has gone $(q - p)$ miles. How long has it taken to do this? Instead of subtracting 2 from 5, we subtract a from b. The car has taken $(b - a)$ hours. To find the velocity, we divide the number of miles gone by the number of hours taken. This gives us

Formula (1)
$$v = \frac{q - p}{b - a}.$$

It is most important to remember that this formula holds only if the car has a steady speed—if it moves at a constant velocity.

Suppose, for example, a car driver drove 30 miles in one hour, then spent 3 hours having dinner, suddenly realized how late it was, drove for an hour at 95 mph, and then had an accident. It would be no good for this driver to say, "I have been out for 5 hours and have covered 125 miles. So my speed can only have been 25 mph. The accident was not my fault." At the moment of the accident, his speedometer was showing 95 mph. That is what we mean by velocity; what the speedometer shows *at a particular instant*. It has nothing to do with ancient history. Maybe this driver had not used his car for a year. Then he could say that he had only covered 125 miles in a year, which is 0.014 miles an hour. Everyone would call this a ridiculous defense. I only emphasize this point because many students of calculus behave exactly like this man. They remember formula (1). It is so simple that they use it even in situations where it gives the most ridiculous results.

Formula (1) works only when an object travels with constant velocity. If the velocity varies a little, then formula (1) gives us, not the exact velocity, but a reasonably close estimate of it. For example, the speed of a car does not vary much in one second. Formula (1) would give *a reasonable estimate* of a car's speed, if one observed the distance the car went in a second. Such evidence might be available if someone had been taking a movie when a car crashed, and it would be quite reasonable to produce that movie in a law court. In calculus, we use something of the same procedure. We are mainly interested in cases where the velocity is varying all the time. So we cannot simply quote formula (1). That would be quite wrong. What we do, is to use formula (1) to *estimate* the velocity; by using shorter and shorter times, we try to arrive at some conclusion.

Negative Velocity

One curious result can be drawn from formula (1) even in the case of steady velocity. Suppose the car is going *backwards*. This happens rarely or never with cars, so our example is somewhat unreal. However, in science the situation frequently occurs; for example, a stone, thrown straight up into the air, rises for a certain time, and then falls. When falling, it is returning to its original position, like a car backing. Suppose then, a car capable of driving backwards at a steady speed for two or three hours. How would its table look? Something like this—

t	3	5
s	80	60

At 3 o'clock, it would be 80 miles from home; at 5 o'clock, only 60 miles. In 2 hours, it has returned 20 miles; evidently, it has been backing at 10 mph.

What does formula (1) give? We have to put

$$a = 3, \qquad b = 5, \qquad p = 80, \qquad q = 60.$$

This gives

$$v = \frac{q - p}{b - a} = \frac{60 - 80}{5 - 3} = \frac{-20}{2} = -10.$$

We know the car is backing at 10 mph. The formula gives $v = -10$.

There are two ways of dealing with this situation.

(1) We might say, "It is absurd to have negative velocities. A velocity cannot be less than zero. If a car is going backwards, you must use a different formula. Formula (1) just does not apply then."

(2) We might say, "We will use formula (1) always when something moves with a steady speed. If formula (1) gives us a negative answer, we shall know that the object is moving backwards."

Policy (2) has been found to be much the most convenient. If we used policy (1), it would double our work; we should have one set of rules for things that are rising, another set for things that are falling. Policy (2) allows us to have a single formula. If, at the end, the answer comes out negative, we know what that means. Usually, in a car, the speedometer shows only speeds *forward*. What we are doing now is rather more like what happens on a ship, where you have "full speed ahead" and "full speed astern." One could imagine a car with an extended speedometer, that went past zero to show "-5 mph" when the car was backing at 5 mph, "-10 mph" when it was backing at 10 mph, and so on.

In physics, the word *velocity* is commonly used when direction is being taken into account; *speed* is used when you are simply concerned with how fast an object is moving, and not bothering whether it is moving forwards or backwards. Thus a car advancing at 10 mph has a *velocity* of $+10$ mph; when backing at 10 mph, it has a *velocity* of -10 mph. In both these cases, the *speed* is 10 mph. This distinction will not play any part in this book. We shall always be concerned with velocity. For example, we might record various movements as in Fig. 8.

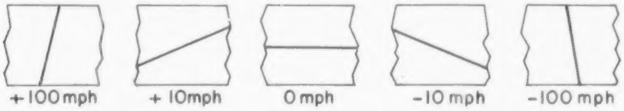


Figure 8

Rates of Change

If we are traveling in a car, the velocity of the car is the rate at which the mileage increases. Velocity is the rate of change of distance gone. Calculus is concerned with how fast things change. The thing changing need not be a distance. We may ask, "How fast is that man growing rich?" "How fast is this car's tank being filled with gas?" These are rates of change—the rate of change of a bank account; the rate of change of the amount of gas in the tank.

It is convenient to have a symbol for "the rate of change of." We shall use a very simple one, the symbol

If f measures any quantity, f' measures the rate at which that quantity is growing (f' is read "f prime" or "f dashed").

For example, if a boy is h inches in height when he is n years old, h' means the rate at which he is growing, in inches a year.

If a car goes s miles in t hours, s' means the rate, in mph, at which the mileage grows. s' miles an hour is in fact the velocity of the car.

If there are g gallons of gas in a tank after t seconds of filling, g' means the rate at which gas is entering the tank, measured in gallons a second.

If a man has m dollars when he is n years old, m' is the rate at which his wealth is increasing, in dollars a year.

Note here the distinction we made earlier: m' is not the same as m/n. If a man has $3000 when he is 30 years old, it does not in the least follow that his wealth is increasing at the rate of $100 a year. You could only draw this conclusion if you knew that, from the time he was born, he had been saving money at a steady rate. It might be that he had nothing at all until he was 27, and in the last three years he has been saving steadily at $1000 a year. In that case, m' would be 1000. On the other hand, it may be that he is having a difficult time now, and is actually losing money at $500 a year. In that case $m' = -500$. m' has nothing to do with ancient history. It measures what is happening now.

If s miles is the distance a car has gone in t hours, s' denotes the velocity of the car in miles an hour. Again, you cannot assume that $s' = s/t$. If I tell you that I have been driving for 3 hours and have covered 90 miles, you cannot work out from this how fast I am moving *at this moment*. You can only see what s' is by looking at the speedometer. I may be traveling at sixty. In this case, $s = 90$, $t = 3$, $s' = 60$. Or my car may be at rest. In that case $s = 90$, $t = 3$, $s' = 0$. I may even be backing at 10 miles an hour. Then $s = 90$, $t = 3$, $s' = -10$.

All this merely amounts to saying that, if I tell you what time it is and where I am, you cannot tell me how fast I am moving. However it is necessary to emphasize this. Students seem to have had drilled into them "velocity is distance divided by time." This is so *only in the case of steady velocity*. But the whole point of calculus is to study variable velocity, as when a ball is falling to the earth or a rocket taking off from the earth.

s' then is the number to which the speedometer is pointing at any particular moment.

EXAMPLES. Translate into calculus symbolism:

(1) After I had been traveling for 5 hours, I had covered 120 miles and was driving at 40 mph.

ANSWER. For $t = 5$, $s = 120$ and $s' = 40$.

(2) After 2 hours' driving, my speedometer showed 50 mph and after 3 hours it showed 45 mph.

ANSWER. For $t = 2$, $s' = 50$. For $t = 3$, $s' = 45$.

(3) For the first two hours, I drove at a steady speed of 40 mph.

ANSWER. $s' = 40$ for every value of t from 0 to 2.

Finding Velocity in Simple Cases

There are some cases in which velocity can be found by arithmetic alone. These cases are, of course, not very interesting or exciting; the interesting results come in the problems where new methods are needed. These simple cases, however, can get us used to the s' symbolism.

Suppose the mileage on my car is zero, and I drive at a steady velocity of 10 mph for a certain time. The table giving my mileage at any time is

t	0	1	2	3	4
s	0	10	20	30	40

Here, $s = 10t$ is the law. What is s'? We said at the outset that my velocity was steady at 10 mph, and s' measures my velocity. So $s' = 10$. Let us set this out formally.

Result A. If

$$s = 10t,$$

$$s' = 10.$$

Since my velocity is 10 mph all the time, $s' = 10$ does not simply mean that s' is 10 at some particular instant, but that *at any instant* during the motion s' has the value 10. $s = 10t$ is a law for the motion in the sense that it tells you where the car is at any time. If you ask, "Where is the car after $1\frac{1}{2}$ hours?" I substitute $t = 1\frac{1}{2}$ in the formula $s = 10t$ and get $s = 15$. $s' = 10$ is also a law, in the sense that it tells me the velocity at any time; it says that the velocity is always 10.

Here we have an example of one of the first problems of calculus: given a law that tells you where an object is at any time, find a law for its velocity at any time.

Exercises

1. To begin with, the mileage of my car is zero. I drive at a steady velocity of 20 mph. What law gives my position at any time? What is my velocity at any time? Write the answers to both questions as equations.
2. The position of a car at any time is given by the equation $s = 30t$. What is the mileage when $t = 0$? when $t = 1$? when $t = 2$? when $t = 3$? What is the velocity of the car? What equation gives s'?
3. The position of a car at any time is given by the equation $s = 40t$. Find the equation for the velocity of the car.
4. Complete the statement, "if $s = 50t$, $s' = \ldots$".
5. If k stands for any fixed number (like 20, 30, 40, 50 in the preceding examples) and $s = kt$, then $s' = \ldots$?

In the examples just considered, we started each time with zero mileage. This however is not necessary. Consider the law $s = 10t + 3$. The table for this is

t	0	1	2	3	4
s	3	13	23	33	43 .

Here, the mileage recorder showed 3 at the beginning. The table shows that the car covers 10 miles with every hour that passes. The velocity is 10 mph, and so $s' = 10$. We thus have

Result B. If

$$s = 10t + 3,$$
$$s' = 10.$$

Speed

Velocity at an Instant

Steady velocity is too simple to be very exciting. We now turn to the real problem, the question of variable velocity.

It should be emphasized that the quantity v or s', for which we are seeking, is intended to measure velocity *at an instant*. In everyday life we find this quite simple; we glance at the speedometer of a car; the needle points to 60 mph and we conclude that 60 mph is our speed at this instant. But when we start to examine what this means, we meet a certain paradox. The very idea of velocity seems to involve *two* times, the beginning and end of an interval. We measure velocity in miles an hour, and these words imply that we see how far an object goes in a certain time. If the time allowed is zero, the distance the object goes is zero. However fast it may be going, two photographs of it taken at the same time will show it at the same place.

If in formula (1) we were to try to discover the velocity at an instant, by making a and b coincide, then p and q also would coincide, and the formula would give us $0 \div 0$ as the velocity—which does not help us at all.

We have used curves to record the movement of objects. A steep line corresponds to an object moving fast; a gentle slope to an object moving slowly (Figs. 5 and 6). So our question could be posed in terms of curves. Instead of saying, "What is the velocity at this instant?" we could ask, "What is the steepness of the curve at the point P?" (see Fig. 9). This seems a sensible sort of question. We would agree, for

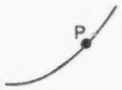


Figure 9

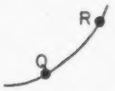


Figure 10

example, that, for the curve shown in Fig. 10, the steepness at the point *R* is greater than at the point *Q*. We know what we mean when we say this. But suppose the curve were covered up in such a way that we could only see the point *Q* itself (Fig. 11). We should have no idea how steep the curve was at *Q*. Suppose the screens are moved a little apart, so that we see just a little bit of the curve near *Q* (Fig. 12).

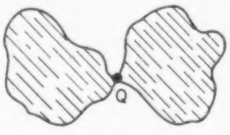


Figure 11

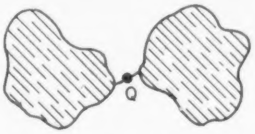


Figure 12

Now we can see what the steepness is at *Q*; it does not matter how little of the curve is exposed, so long as we can see a piece of curve on each side of *Q*.

Accelerated Motion

Let us now take a particular case of motion with variable velocity, and see how the velocity at any instant can be calculated. This example that we are going to study is in fact of importance in physics; it is the type of motion usually studied at the beginning of a course in mechanics. It could be produced by the apparatus shown in Fig. 13.

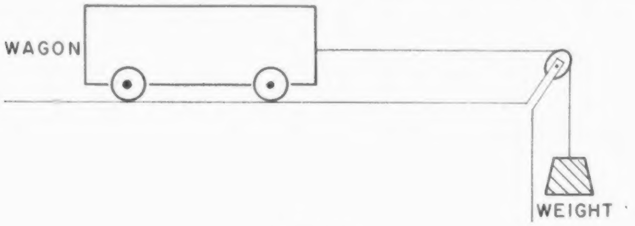


Figure 13

If the wagon weighed 15 ounces, the weight would have to be somewhat more than 1 ounce. "Somewhat more" because there would be friction acting at the wheels of the wagon; by adjusting the weight, the desired motion could be obtained, namely, that given by the table

t	0	1	2	3	4
s	0	1	4	9	16

.

It is understood that s feet is the distance gone by the wagon in t seconds. The table of course fits the law

$$s = t^2.$$

You will notice that the table above agrees with my statement that we have accelerated motion. In the first second, between $t = 0$ and $t = 1$, the wagon advances 1 foot only. But between $t = 1$ and $t = 2$, the wagon advances 3 feet. Between $t = 2$ and $t = 3$, the wagon advances 5 feet (for $5 = 9 - 4$). Between $t = 3$ and $t = 4$, the wagon advances 7 feet ($7 = 16 - 9$). These numbers are consistent with the belief that the wagon is accelerating, is going faster and faster, as the weight pulls it forward.

Suppose now we try to estimate the velocity at the instant when $t = 3$. In the second before this instant, from $t = 2$ to $t = 3$, the wagon covers 5 feet. In the second after this instant, from $t = 3$ to $t = 4$, the wagon covers 7 feet. It seems reasonable to guess that the velocity at the instant $t = 3$ lies between 5 and 7 feet a second.

Students nearly always ask, "Couldn't we take the average of 5 and 7, and say that the velocity is 6 feet a second?" Unfortunately, this answer is correct for this particular example. I say, "Unfortunately," because, as a rule, taking the average does not give the correct velocity. In fact it hardly ever gives the correct velocity. Only when the law is of the type

$$s = at^2 + bt + c$$

does taking the average work. We shall see below that averaging gives a wrong result for the law $s = t^3$.

If you will take my word for this, for the time being, we shall set aside the guess that the true velocity is exactly halfway between our estimates 5 and 7, and merely use our conclusion that the velocity lies *somewhere* between 5 and 7.

How can we narrow down this margin? We agreed earlier that the shorter the time interval was, the better estimate one should get for the velocity. It seems a good idea to take a shorter interval. Instead of one second before and after $t = 3$, we try half a second before and after. By substituting in the formula $s = t^2$, we obtain the little table

t	$2\frac{1}{2}$	3	$3\frac{1}{2}$
s	$6\frac{1}{4}$	9	$12\frac{1}{4}$

.

What use has the wagon made of these half seconds? In the half second between $t = 2\frac{1}{2}$ and $t = 3$, s has grown from $6\frac{1}{4}$ to 9. That is, the wagon has covered $2\frac{3}{4}$ feet. Two and three-quarters feet in $\frac{1}{2}$ second suggests a velocity of $2\frac{3}{4} \div \frac{1}{2}$, which equals $5\frac{1}{2}$ feet a second.

In the half second after $t = 3$, the wagon covers $12\frac{1}{4} - 9$, that is, $3\frac{1}{4}$ feet. Three and one-quarter feet in $\frac{1}{2}$ second suggests the velocity $3\frac{1}{4} \div \frac{1}{2}$, that is, $6\frac{1}{2}$ feet a second.

So we now think the velocity should lie between $5\frac{1}{2}$ and $6\frac{1}{2}$ feet a second.

But why stop at a half? Why not go to shorter and shorter intervals, getting better and better estimates?

If we take one-tenth of a second before and after $t = 3$, we get the little table

t	2.9	3	3.1
s	8.41	9	9.61

by means of the formula $s = t^2$. In the tenth of a second before $t = 3$, the wagon advances 0.59 feet; this suggests a velocity of $0.59 \div 0.1 = 5.9$ feet a second. In the tenth of a second after $t = 3$, the wagon advances 0.61 feet, which suggests a velocity of $0.61 \div 0.1 = 6.1$ feet a second. We now think the velocity should lie between 5.9 and 6.1 feet a second.

By exactly the same method, if we take one-hundredth of a second before and after $t = 3$, we are led to believe that the velocity lies between 5.99 and 6.01 feet a second. By taking one-thousandth of a second, we are led to believe the velocity is between 5.999 and 6.001 feet a second.

We collect these results in the form of a table:

By considering intervals of	We are led to believe that v, the velocity in feet per second, lies between
1 second	5 and 7
0.1 second	5.9 and 6.1
0.01 second	5.99 and 6.01
0.001 second	5.999 and 6.001

Our last estimate here, using 0.001 second, pins v down to a very narrow region, since 5.999 and 6.001 differ by only 0.002. But of course there is no need to stop at an interval of 0.001 second. We could use a millionth or a billionth of a second, and get even more accurate estimates of v. In fact, there seems to be no limit to how accurately we can estimate v. For the table above shows a very marked pattern. I should imagine you can guess how the table would continue.

Exercise

Without making any calculations, guess the estimates of v that would correspond to intervals of 0.0001 second and 0.00001 second. Check your guesses by actual calculation.

I imagine you had no difficulty in seeing how the table would continue. Each row we go down, we find one more 9 in 5.99 . . . 9 and one more zero in 6.00 . . . 01. The estimates are coming closer and closer together. Any particular estimate leaves some uncertainty about the value of v, even though this uncertainty may be very small. But if we take *all* the estimates into account, this uncertainty disappears. There is only one number that is bigger than 5.999 . . . 9, however many nines are written, and smaller than 6.000 . . . 01, however many zeros are written. That number is 6.

So, although we spoke of estimating the velocity, and an estimate usually implies some degree of error or uncertainty, yet there is no uncertainty at all in our final answer. 6 is the only number that satisfies all the estimates, as they close in from the right and the left.

All this arithmetic thus leads us to the conclusion that, if a body moves according to the law $s = t^2$, when $t = 3$ its velocity is given by $v = 6$.

The purpose of this explanation is that you should now be able to work out for yourself the values of v corresponding to $t = 1$, $t = 2$, $t = 4$, and $t = 5$. When you look at your answers you should notice a certain law.

I must make sure that you understand the method for finding v corresponding to any given value of t. In classes, some students see the point of the method straight away; but there are always some who have to have it explained more than once. So, for readers who need it, I will indicate how to get clear about the method. It is important that you should understand this method, for the next stage of the work requires you to discover the first result of calculus; you will feel much happier and more confident if you discover it for yourself, than if I have to tell you.

First of all, you must be clear as to the idea behind the method. Formula (1), which is often spoken in the form "velocity is distance divided by time," applies only to constant velocities. When the velocity is varying, distance divided by time gives the *average velocity* only; the actual velocity at any instant may be more or less than the average velocity. However, we consider shorter and shorter intervals of time; we hope that this gives less and less opportunity for the velocity to vary, so that the average velocity, over a very short interval, should be a good estimate of the true velocity.

Second, you need to be able to carry through the actual calculation. If you find difficulty in organizing the work, you may find it helpful to adapt the argument of pages 24–25; go through the same kind of steps, but work out the velocity for $t = 2$ instead of $t = 3$. Then go through the steps again, but this time find v for $t = 4$. Of course, do not be content just to go through the arithmetic. Think all the time what you are doing and why that should be done.

When you have worked out v corresponding to $t = 1$, $t = 2$, $t = 4$ and $t = 5$, complete the following table:

t	1	2	3	4	5
v	......		6	......	.

After completing the table, you should notice a law connecting v and t. The law is $v =$ ....

It is best if you do not read further until you have successfully completed this work.

* * *

The Law for the Velocity

If you carry through the arithmetic correctly, you should arrive at the following result:

t	1	2	3	4	5
v	2	4	6	8	10 .

Each number in the second row is exactly twice the number above it. So the law is $v = 2t$. If we use the sign $'$ introduced on page 18, we may use s' instead of v. We then have a new result to put beside our results A and B on pages 19, 20.

Result C. If

$$s = t^2,$$

$$s' = 2t.$$

This very simple result has come out of long calculations. We shall in a moment consider how it might be found more shortly. However, these long calculations have in no way been wasted. This work should have given you a feeling for what is happening. Many students read short proofs of this result in calculus texts; they pass quickly over the algebra, and they never realize what it really means.

Let us now see how algebra could be used to reduce our work, and also to make the argument more convincing.

When we considered how far the object went according to the law $s = t^2$ between the times $t = 2.99$ and 3, we had the messy arithmetical task of finding the square of 2.99. This work can be simplified by algebra. There is a standard result of algebra

Formula (2) $$(a + b)^2 = a^2 + 2ab + b^2.$$

If we put $a = 3$, $b = -0.01$, we get $a + b = 2.99$. So formula (2) gives us

$$\begin{aligned}(2.99)^2 &= 3^2 + 2 \cdot 3 \cdot (-0.01) + (0.01)^2\\ &= 9 - 0.06 + 0.0001\\ &= 8.9401.\end{aligned}$$

This method involves less work, and is less likely to lead to a mistake than the usual method of elementary arithmetic.

However, we can make greater use of algebra than simply to shorten the calculations. On page 25, we observed a column containing the numbers 5; 5.9; 5.99; 5.999; and we made a guess as to how this column would continue. By using algebraic symbols, we can avoid this guess. Instead of considering, one at a time, the intervals

between 3 and $3 + 0.1$;
between 3 and $3 - 0.1$;
between 3 and $3 + 0.01$;
between 3 and $3 - 0.01$; etc.,

we can notice that all these intervals are particular cases of the interval

between 3 and $3 + h$.

The particular cases can be got by substituting 0.1; -0.1; 0.01; -0.01; respectively for h. Since we can equally well substitute 0.0000001 or 0.000000001 for h, we are thus enabled to deal with the intervals of one-millionth or one-billionth, or any other number for that matter, all at one blow, by an algebraic calculation.

We now carry this idea into practice. We want to find the velocity at $t = 3$. So we consider a short interval, from $t = 3$ to $t = 3 + h$. We must find where the object is at these times. The position is determined by the formula $s = t^2$. When $t = 3$, $s = 9$. When $t = 3 + h$, $s = (3 + h)^2 = 9 + 6h + h^2$. So we have the table

t	3	$3 + h$
s	9	$9 + 6h + h^2$.

We now use "distance divided by time" to estimate the velocity. How far has the object gone during this interval of time? We take the difference between the numbers in the row giving s. The object has gone $6h + h^2$ feet during the interval. How long is the interval of time? We take the difference between the numbers for t. The interval is of length h seconds. Division gives us our estimate of v, namely,

$$\frac{6h + h^2}{h}.$$

The expression can be simplified. Since

$$6h + h^2 = h \cdot (6 + h),$$

on dividing both sides by h we have

$$\frac{6h + h^2}{h} = 6 + h.$$

Exercise

In the above expression substitute in turn, the values 1; 0.1; 0.01; −1; −0.1; −0.01 for h, and check that the results agree with numbers in the table on page 25, the positive values of h giving one column and the negative values of h the other.

When h is positive, we are considering a little interval just after $t = 3$. Our estimate of v is then $6 + h$, just a little more than 6.

When h is negative, we are considering a little interval just before $t = 3$. Our estimate of v is then just a little less than 6. (For example, if $h = -0.01$, the estimate $6 + h$ is $6 + (-0.01)$, that is 5.99, just less than 6.)

The shorter we make the interval, the closer our estimate comes to 6. We are thus led to the conclusion that $v = 6$.

We have just found, by algebra, the velocity corresponding to $t = 3$. Now there is nothing special about the number 3. The work would have been just as easy for any other number. Here again is the sort of situation where algebra can help; we can use a symbol for "any number" and find v corresponding to any value of t.

Suppose then, we try to find the velocity when $t = a$, where a stands for "any number." The work will follow exactly the same plan as it did for $t = 3$. We can go through this work, step by step, but writing a wherever 3 came before.

Exercise

Do this, if you can, before reading it below.

* * *

We shall have the table

t	a	$a + h$
s	a^2	$a^2 + 2ah + h^2$.

Distance gone during the interval is found from the difference between the two numbers in the row for s; the distance is $2ah + h^2$ feet. The time taken is found from the difference between the numbers in the row for t. The time taken is h seconds. Division gives the estimate

$$\frac{2ah + h^2}{h}$$

for v, and this simplifies to

$$2a + h.$$

Now h is a very small number; it represents the length in seconds of the interval during which we observe the motion; the shorter this interval, the better the estimate of v. As h gets smaller and smaller, $2a + h$ approaches $2a$. We conclude

$$v = 2a.$$

Thus, if $t = a$, $v = 2a$. In words, "If t is any number, v is twice that number." This confirms the guess we made on page 27. But there our evidence was limited to the numbers 1, 2, 3, 4, 5. By using algebra, we see that $v = 2t$ holds for *every* value of t. Of course a need not be a whole number; the laws of algebra hold equally well for fractions and irrational numbers.

You may wonder why we bother to bring the number a into the discussion. Why write $t = a$, $v = 2a$ instead of just $v = 2t$? The reason is that, at the beginning of our work we had to consider an interval of h seconds, from $t = a$ to $t = a + h$. If we had tried to do without a, we might have found ourselves talking about the interval from $t = t$ to $t = t + h$, which sounds somewhat peculiar!

A Useful Symbolism

In discussing motion, we continually use phrases such as "where the body is at a certain time," or the corresponding algebraic phrase, "the value of s corresponding to a particular value of t." Seeing this phrase is used so often, it is convenient to have an abbreviation for it. We shall use $s(a)$ to stand for "the value of s corresponding to $t = a$." Thus, in the table

t	0	1	2	3
s	0	1	4	9

9 is the value of s corresponding to $t = 3$; we can save a lot of space by expressing this in the abbreviated form $s(3) = 9$. For the same table, $s(0) = 0$, $s(1) = 1$, $s(2) = 4$.

When we are discussing velocities, we consider the interval of time from $t = a$ to $t = a + h$. We then examine where the object is at the beginning and end of this interval. Its position is specified by the value of s. The value of s corresponding to $t = a$ can now be indicated by $s(a)$, and the value of s corresponding to $t = a + h$ by $s(a + h)$.

Thus, in this interval, the object covers a distance of $s(a + h) - s(a)$ feet. The time taken is $(a + h) - a = h$ seconds. Thus the average velocity during the interval is

Formula (3)
$$\frac{s(a + h) - s(a)}{h}$$

feet per second.

Procedure for Determining Velocity

We are now able to describe the steps by which we found the law for velocity in our work above. The purpose of describing the procedure is, of course, so that we can apply it to other laws besides $s = t^2$.

(1) We began with a law giving s in terms of t.

(2) We then considered the average velocity during the interval between $t = a$ and $t = a + h$. This led us to the expression given in formula (3) above, namely,

$$\frac{s(a + h) - s(a)}{h} .$$

(3) We allowed h to become smaller and smaller. Thus h approached the value zero. We then found that

$$\frac{s(a + h) - s(a)}{h}$$

approached a certain value.

(4) That value we regarded as giving the velocity at the instant $t = a$.

In our symbolism, this result would be written $v(a)$ or $s'(a)$, for it gives the value of v or s' at $t = a$.

Clifford was an innovative mathematician and physicist at the end of the nineteenth century. This geometrical introduction to kinematics, completed by Karl Pearson after Clifford's death, uses very little algebra.

10

Motion

William Kingdon Clifford

1879

§1. *On the Various Kinds of Motion*

WHILE the chapters on Space and Position considered the sizes, the shapes, and the distances of things, the present chapter on Motion will treat of the changes in these sizes, shapes, and distances, which take place from time to time.

The difference between the ordinary meaning attached to the word "change" in everyday life and the meaning it has in the exact sciences is perhaps better illustrated by the subject of this chapter than by any other that we have yet studied. We attained exactness in the description of quantity and position by substituting the method of representing them by straight lines drawn on paper for the method of representing them by means of numbers; though this, at first sight, might easily seem to be a step backwards rather than a step forwards, since it is more like a child's sign of opening its arms to show that its stick is so long, than a process of scientific calculation.

It is, however, by no means an easy thing to give an accurate description of motion, even although it is itself as common and familiar a conception as quantity or position.

Let us take a simple case. Suppose that a man, on a railway journey, is sitting at one end of a compartment with his face towards the engine; and that, while the train is going along, he gets up and goes to the other end of the compartment and sits down with his back to the engine. For ordinary purposes this description is amply sufficient, but it is very far indeed from being an exact description of the motion of the man during that time. In the first place, the train was moving, and it is necessary to state in what direction, and how fast it was going at every instant during the interval considered. Next, we must describe the motion of the man relatively to the train; and, for this purpose, we must neglect the motion of the train and consider how the man would have moved if the train had been at rest. First of all, he changes his position from one corner of the compartment to the opposite corner; next, in doing this he turns round; and, lastly, as he is walking along or rising up or sitting down, the size and shape of many of his muscles are altered. We should thus have to say, first, exactly how fast and in what direction he was moving at every instant, as we had to do in the case of the train; then, how quickly he was turning round; and, lastly, what changes of size or shape were taking place in his muscles, and how fast they were occurring.

It may be urged that this would be a very troublesome operation, and that nobody wants to describe the motion of the man so exactly. This is quite true; the case which has been taken for illustration is not one which it is necessary to describe exactly, but we can easily find another case which is very analogous to this, and which it is most important to describe exactly. The earth moves round the sun once in every year; it is also rotating on its own axis once every day; the floating parts of it—the ocean and the air—are constantly undergoing changes of shape and state which we can observe and which it is of the utmost importance that we should be able to predict and calculate; even the solid nucleus of the earth is constantly subject to slight changes in size and shape, which, however, are not large enough to admit of accurate observation. Here, then, is a problem whose complexity is quite as great as that of the former, and whose solution is of pressing practical importance.

The method which is adopted for attacking this problem of the accurate description of motion is to begin with the simplest cases. By the simplest cases we mean those in which certain complicating circumstances do not arise. We may first of all restrict ourselves to the study of the motions of these bodies in which there is no change of size or shape. A body which preserves its size and shape unaltered during the interval of time considered is called a *rigid* body. The word "rigid" is here used in a technical sense belonging to the science of dynamic, and does not mean, as in ordinary language, a body which resists alteration of size and shape, but merely a body which, during a certain time, happens not to be altered in those respects. Then, as the first and simplest case, we should study that motion of a rigid body in which there is no turning round, and in which therefore every line in the body keeps the same direction (though of course not the same position) throughout the motion. We state this by saying that every line "rigidly connected" with the body remains parallel to itself. Such a motion is called a *motion of translation*, or simply a *translation*; and so the first and simplest case we have to study is the translation of rigid bodies. After that we must proceed to consider their turning round, or *rotation*; and then we have to describe the changes of size or shape which bodies may undergo, these last changes being called *strains*. The study of motion therefore requires the further study of translations, of rotations, and of strains, and further, the art of combining these together. When we have studied all this we shall be able to describe motions exactly; and then, but not till then, will it be possible to state the exact circumstances under which motions of a given kind occur. The exact circumstances under which motions of a given kind occur we call a *law of nature.*

§2. *Translation and the Curve of Positions*

Let us talk, to begin with, of the translation of a rigid body.

Suppose a table to be taken from the top to the bottom of a house in such a manner that the surface of it is always kept horizontal, and that its length is made always to point due north and south; it may be taken down a staircase of any form, but it is not to be turned round or tilted up. The table will then undergo a translation. If we now consider a particular corner of the table, or the end of one of its legs, or any other point, this point will have described a certain curve in a certain manner; that is to say, at every point of this curve it will have been going at a certain definite rate. Now the important property of a motion of translation, which makes it more easy to deal with than any other motion, is that for all points of the body this curve is the same in size and shape and mode of description. That this is so in the case of the table is at once seen from the fact that the table is never turned round nor tilted up during the motion, so that the different points of it must at any instant be moving in the same direction and at the same rate. In order therefore to describe this motion of the table it will be sufficient to describe the motion of any point of it, say the end of one of its legs. And so, in general, the problem of describing the motion of translation of any rigid body is reduced to the problem of describing the motion of a point along a curve.

Now this is a very much easier task than our original problem of describing the motion of the earth or the motion of the man in the train; but we shall see that, by properly studying this, it will be easy to build up out of it other more complicated cases. Still, even in this form our problem is not quite simple enough to be directly attacked. What we have to do, it must be remembered, is to state exactly where a certain point was, and how fast it was going at every instant of time during a certain interval. This would require us first to describe exactly the shape of the curve along which the point moved; next, to say now far it had travelled along the curve from the beginning up to any given instant; and lastly, how fast it was going at that instant. To deal with this problem we must first take the very simplest case of it, that, namely, in which the point moves along a straight line, and leave for the present out of account any description of the rate of motion of the point; so that we have only to say where the point was on a certain straight line at every instant of time within a given interval.

But we have already considered what is the best way of describing the position of a point upon a straight line. It is described by means of the step which is required to carry it to that position from a certain standard place, viz. a step from that place so far to the right or to the left. To specify the length of the step, if we are to describe it exactly, we must not make use of any words or numbers, but must draw a line which will represent the length corresponding to every instant of time within a certain interval, so that we may always be able to answer the question, Where was the point at this particular instant? But a question, in order to be exactly answered, must first be exactly asked; and to do this it is necessary that the instant of time about which the question is asked should be accurately specified.

Now time, like length, is a continuous quantity which cannot in general be described by words or numbers, but can be by the drawing of a line which shall represent it to a certain scale. Suppose, then, that the interval of time during which the motion of a point has to be described is the interval from twelve o'clock to one o'clock. We must mark on a straight line a point to represent twelve o'clock and another point to represent one o'clock; then every instant between twelve o'clock and one o'clock will be represented by a point which divides the distance between these two marked points in the same ratio in which that instant divides the interval between twelve o'clock and one o'clock. Then for every one of these points it is necessary to assign a certain length, representing (to some definite scale) the distance which the point has travelled up to that instant; and the question arises, In what way shall we mark down these lengths?

Let us first of all observe the difficulty of answering this question. If we could be content with an approximate solution instead of an exact one, we might make a table and put down in inches and decimals of an inch the distances travelled, making an entry for every minute, or even perhaps for every second during the hour. Such tables are in fact constructed and published in the "Nautical Almanac" for the positions of the moon and of the planets. The labour of making this table will evidently depend upon its degree of minuteness; it will of course take sixty times as long to make a table showing the position of the point at every second as to make one showing the position at every minute, because there will be sixty times as many values to calculate. But the problem of describing exactly the motion of the point requires us to make a table showing the position of the point at every instant; that is, a table in which are entered an infinite number of values. These values moreover are to be shown, not in inches and decimals of an inch, but by lengths drawn upon paper. Yet we shall find that this pictorial mode of constructing the table is in most cases very much easier than the other. We have only to decide where we shall put the straight lines which represent the distances that the point has travelled at different instants.

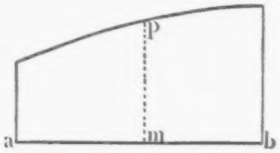


Fig. 113

Let *ab* (Fig. 113) be the length which represents the interval of time from twelve o'clock to one o'clock, and let *m* be the point representing any intermediate instant. Then if we draw at *m* a line perpendicular to *ab* whose length shall represent (to any scale that we may choose) the distance that the point has up to this instant travelled, then *p*, the extremity of this line, will correspond to an entry in our table. But if such lines be drawn perpendicular to *ab* from every point in it, all the points *p*, which are the several extremities of these lines, will lie upon some curve; and this curve will represent an infinite number of entries in our table. For, when once the curve is drawn, if a question is asked: What was the position of the point at any instant between twelve o'clock and one o'clock? (this instant being specified in the right way by marking a point between *a* and *b* which divides that line in the same ratio as the given instant divides the hour), then the answer to this question is obtained simply by drawing a line through the marked point perpendicular to *ab* until it meets the curve; and the length of that line will represent, to the scale previously agreed upon, the distance travelled by the point.

Such a curve is called the *curve of positions* for a given motion of the point; and we arrive at this result, that the proper way of specifying exactly a translation along a straight line is to draw the curve of positions.

We have now learned to specify, by means of a curve, the positions of a body which has motion of translation along a straight line; and we have not only represented an infinite number of positions instead of a finite number, which is all a numerical table would admit, but have also represented each position with absolute exactness instead of approximately. It is important to notice that in this and in all similar cases the exactness is ideal and not practical; it is exactness of conception and not of actual measurement. For though it is not possible to measure a given length and to state that measure any more accurately by drawing a line than it is by writing it down in inches and decimals of an inch, yet the representation by means of a line enables us to reason upon it with an exactness which would be impossible if we were restricted to numerical measurement.

§3. *Uniform Motion*

Hitherto we have supposed our point to be moving along a straight line, but were it to move along a curve the construction given for the curve of positions would still hold good, only the distance traversed at any instant must now be measured from some standard position *along the curve*. Hence any motion of a point, or any motion of translation whatever, can be specified by a properly drawn curve of positions, and the problem of comparing and classifying different motions is therefore reduced to the problem of comparing and classifying curves. Here again it is advisable and even necessary to begin with a simple case. Let us take the case of uniform motion, in which the body passes over equal distances in equal times; and then, as we may easily see, the curve of positions is a straight line. Uniform motion may also be described as that in which a body always goes at the same rate, and not quicker at one time and slower at another. It is obvious that in this case any two equal distances would require equal times for traversing them, so that the two descriptions of uniform motion are equivalent.

It was shown by Archimedes (the proof is an easy one, depending upon the definition of the fourth proportional) that whenever equal distances are traversed in equal times, different distances will be traversed in times proportional to them. Assuming this proposition, it becomes clear that the curve of positions must be a straight line, for a straight line is the only curve which has the property that the height of every point of it is proportional to its horizontal distance from a fixed straight line.

We may also see in the following manner the connection between the straight line and uniform motion.

Suppose we walk up a hill so as always to get over a horizontal distance of four miles in an hour. The rate at which we go up will clearly depend on the steepness of the hill; and if the hill is a plane, *i.e.* is of the same steepness all the way up, then our rate of ascent will be the same at every instant, or our upward motion will be uniform. If the hill be four miles long and one mile high, then, since the four miles of horizontal distance will be traversed in an hour, the one mile of vertical distance will also be traversed in an hour, and we shall be gaining height at the uniform rate of one mile an hour. If the hill were two miles high, or, as we say twice as steep, then we should have been gaining height at the rate of two miles an hour. But now if we suppose a hill of varying steepness, so that the outline of it seen from one side is a curve, then it is clear that the rate at which we go up will depend upon the part of the hill where we are, assuming that the rate at which we go forward horizontally remains always the same. This "elevation" of the hill may be taken as the curve of positions for our vertical motion; for the horizontal distance that we have gone over, being always proportional to the time, may be taken to represent the time, and then the curve will have been constructed according to our rule, viz. a horizontal distance will have been taken proportional to the time elapsed, and from the end of this line a perpendicular will have been raised indicating the height which we have risen in that time. Uniform motion then has for its curve of positions a straight line, and the rate of the motion depends on the steepness of the line. Variable motion, on the other hand, has a curved line for its curve of positions, and the rate of motion depends upon its varying steepness.

In the case of uniform motion it is very easy indeed to understand what we mean by the rate of the motion. Thus, if a man walks uniformly six miles an hour, we know that he walks a mile in ten minutes, and the tenth part of a mile in one minute, and so on in proportion. It may not, however, be possible to specify this rate by means of numbers; that is to say, the man may not walk any definite number of miles in the hour, and the exact distance that he walks may not be capable of representation in terms of miles and fractions of a mile. In that case we shall have to represent the velocity or rate at which the man walks in much the same way as we have represented other continuous quantities. We must draw to scale upon paper a line representing the length that he has walked in an hour, or a minute, or any other interval of time that we decide to select; thus, for example, a uniform rate of walking might be specified by marking points corresponding to particular hours upon an

Ordnance map. The rate of motion, or *velocity*, is then a continuous quantity which can be exactly specified, as we specify other continuous quantities, but which can be only approximately described by means of numbers.

§4. *Variable Motion*

Let us now suppose that the motion is not uniform, and inquire what is meant in that case by the rate at which a body moves.

A train, for example, starts from a station and in the course of a few minutes gets up to a speed of 30 miles an hour. It began by being at rest, and it ends by having this large velocity. What has happened to it in the meantime? We can understand already in a rough sort of way what is meant by saying that at a certain time between the two moments the train must have been going at 15 miles an hour, or at any other intermediate rate; but let us endeavour to make this conception a little more exact. Suppose, then, that a second train, which is indefinitely long, is moving in the same direction at a uniform rate of 15 miles an hour on a pair of rails parallel to that on which the first train moves; thus, when our first train is at rest the second one will appear to move past it at the rate of 15 miles an hour. When the first train starts an observer seated in it will see the second train going apparently rather more slowly than before, but it will still seem to be moving forwards. As the first train gets up its speed, this apparent forward motion will gradually decrease until the second train will appear to be going so slowly that conversation may be held between the two; this will take place when the rate of the first train has amounted to something nearly but not quite equal to 15 miles an hour, which we supposed to be the constant rate of the second train. But as the rate of the first train continues to increase there will come a certain instant at which the second train will appear to stop gaining upon the first and to begin to lose. At that particular instant it will be neither gaining nor losing, but will be going at the same rate; at that particular instant, therefore, we must say that the first train is going at the rate of 15 miles an hour. And it is at that instant only, for the equality of the rates does not last for any fraction of a second, however small; the very instant that the second train appears to stop gaining it also appears to begin losing. The two trains then run exactly together for no distance at all, not even for the smallest fraction of an inch, and yet we have to say that at one particular instant our first train is going at the rate of 15 miles an hour, although it does not continue to go at that rate during the smallest portion of time. There is no way of measuring this instantaneous velocity except that which has just been described of comparing the motion with a uniform motion having that particular velocity.

Upon this we have to make the very important remark that the rate at which a body is going is a property as purely instantaneous as is the precise position which it has at that instant. Thus, if a stone be let fall to the ground, at the moment that it hits the ground it is going at a certain definite rate; and yet at any previous moment it was not going so fast, since it does not move at that rate for the smallest fraction of a second. This consideration is somewhat difficult to grasp thoroughly, and in fact it has led many people to reject altogether the hypothesis of continuity; but still we may be helped somewhat in understanding it by means of our study of the curve of positions, wherein we saw that to a uniform motion corresponds a straight line and that the rate of the motion depends on the steepness of the line.

Let us now suppose a motion in which a body goes at a very slow but uniform rate for the first second, during the next second uniformly but somewhat faster, faster again during the third second, and so on. The curve of positions will then be represented by a series of straight lines becoming steeper and steeper and forming part of a polygon. From a sufficient distance off this polygon will look like a curved line; and if, instead of taking intervals of a second during which the rates of motion are severally considered uniform, we had taken intervals of a tenth of a second, then the polygon would look like a curved line without our going so far away as before. For the shorter the lengths of the sides of our polygon, the more will it look curved, and if the intervals of time are reduced to one-tenth the sides will be only one-tenth as long. The rate at which the body under consideration is moving when it is in the position to which any point of the polygon corresponds, is obtained by prolonging that side of the polygon which passes through the point; the rate will then depend on the steepness of this line, since, where the line is a side of the polygon, it represents the uniform motion which the body has during a certain interval. When the polygon looks like a curve the sides are very short, and any side, being prolonged both ways, will look like a tangent to the curve.

Now in considering the general case of varying motion we should have, instead of the above polygon which looks like a curve, an actual curve; the difference between them being that, if we look at the curve-like polygon with a sufficiently strong microscope, we shall be able to see its angles, but however powerful a microscope we may apply to the curve it will always look like a curve. But there is this property in common, that if we draw a tangent to the curve at any point, then, since the steepness of this tangent will be exactly the same as the steepness of the curve at that particular point, it will give the rate for the motion represented by the curve, just as before the steepness of the prolonged small side of the polygon gave the rate for the motion represented by the polygon. That is to say, the instantaneous velocity of a body in any position may be learnt from its curve of positions by drawing a tangent to this curve at the point corresponding to the position; for the steepness of this tangent will give us the velocity or rate which we want, since the tangent itself corresponds to a uniform motion of the same velocity as that belonging to the given varying motion at the particular instant. From this means of representing the rate we can see how it is that the instantaneous velocity of a body generally belongs to it only at an instant and not for any length of time however short; for the steepness of the curve is continually changing as we go from one part of it to another, and the curve is not straight for any portion of its length however small.

The problem of determining the instantaneous velocity in a given position is therefore reduced to the problem of drawing a tangent to a given curve. We have a sufficiently clear general notion of what is meant by each of these things, but the notion which is sufficient for purposes of ordinary discourse is not sufficient for the purposes of reasoning, and it must therefore be made exact. Just as we had to make our notion of the ratio of two quantities exact by means of a definition of the fourth proportional, or of the equality of two ratios which were expressed in terms of numbers, so here we shall have to make our idea of a velocity exact by expressing it in terms of measurable quantities which do not change.

We have no means of measuring the instantaneous velocity of a moving body; the only thing that we can measure is the space which it traverses in a given interval of time. In the case in which a body is moving uniformly, its instantaneous velocity, being always the same, is completely specified as soon as we know how far the body has gone in a definite time. And, as we have already observed, the result is the same whatever this interval of time may be; the rate of four miles an hour is the same as eight miles in two hours, or two miles in half an hour, or one mile in a quarter of an hour. But if a body be moving with a velocity which is continually changing, the knowledge of how far it has gone in a given interval of time tells us nothing about the instantaneous velocity for any position during that interval. To say, for instance, that a man has travelled a distance of four miles during an hour, does not give us any information about the actual rate at which he was going at any moment during the hour, unless we know that he has been going at a uniform rate. Still we are accustomed to say that in such a case he must have been going *on an average* at the rate of four miles an hour; and, as we shall find it useful to speak of this rate as an "average velocity," its general definition may be given as follows:—

If a body has gone over a certain distance in a certain time its *mean* or *average velocity* is that with which, if it travelled uniformly, it would get over the same distance in the same time.

This mean velocity is very simply represented by the help of the curve of positions. Let *a* and *b* (Fig. 114) be two points on the curve of positions; then the mean velocity between the position represented by *a* and that represented by *b* is given by the steepness of the straight line *a b*. This, moreover, enables us to make some progress towards a method of calculating instantaneous velocity, for we showed that the problem of finding the instantaneous velocity of a body is, in the above method of representation, the problem of drawing a tangent to a curve. Now the mean velocity of a body is defined in terms of quantities which we are already able to measure, for it requires the measurement of an interval of time and of the distance traversed during that interval; and further the *chord* of a curve, *i.e.* the line joining one point of it to another, is a line which we are able to draw. If then we can find some means of passing from the chord of a curve to the tangent, the representation we have adopted will help us to pass from the mean to the instantaneous velocity.

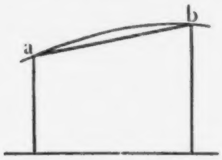


FIG. 114

§5. *On the Tangent to a Curve*

Now let us suppose the chord *a b* (Fig. 115) joining the points on the curve to turn round the point *a*, which remains fixed; then *b* will travel along the curve towards *a*; and if we suppose *b* not to stop in this motion until it has got beyond *a* to a point such as *b′* on the other side, the chord will have turned round into the position *a b′*. Now, looking at the curve which is drawn in the figure, we see that the tangent to the curve at *a* obviously lies between *a b* and *b′ a*. Thus if *a b* turn round *a* so as to move into the position *a b′* it will at some instant have to pass over the position of the tangent. At the instant when it passes over this position where is the point *b*? We can at once see from the figure that it cannot be anywhere else than at *a*, and yet we cannot attach any definite meaning to a line described as joining two coincident points. If we could, the determination of the tangent would be very easy, for in order to draw the tangent to the curve at *a*, we should merely say, Take any other point *b* on the curve; join *a b* by a straight line; then make *b* travel along the curve towards *a*, and the position of the line *a b* when *b* has got to *a* is that of the tangent at *a*. Here however arises the difficulty which we have already pointed out, namely, that we cannot form any distinct conception of a line joining two coincident points; two separate points are necessary in order to fix a straight line. But it is clear that, although it is not yet satisfactory, there is still something in the definition that is useful and correct; for if we make the chord turn from the position *a b* to the position of the tangent at *a*, the point *b* does during this motion move along the curve up to the point *a*.

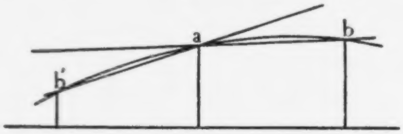


FIG. 115

This difficulty was first cleared up and its explanation made a matter of common sense by Newton. The nature of his explanation is as follows:—Let us for simplicity take the curve to be a circle. If a straight stick be taken and bent so as to become part of a circle, the size of this circle will depend upon the amount of bending. The stick may be bent completely round until the ends meet, and then it will make a very small circle; or it may be bent very slightly indeed, and then it will become part of a very large circle. Now, conversely, suppose that we begin with a small circle, and, holding it fast at one point, make it get larger and larger, so that the piece we have hold of gets less and less bent; then, as the circle becomes extremely large, any small portion of it will more and more nearly approximate to a straight line. Hence a circle possesses this property, that the more it is magnified the straighter it becomes; this property likewise belongs to all the curves which we require to consider. It is sometimes expressed by saying that the curve is straight in its elements, or in its smallest parts; but the statement must be understood to mean only this, that the smaller the piece of a curve is taken the straighter it will look when magnified to a given length.

Now let us apply this to the problem of determining the position of a tangent. Let us suppose the tangent *a t* of a circle to be already drawn, and that a certain convenient length is marked off upon it (Fig. 116); from the end of this T let a perpendicular be drawn to meet the circle in B, and let *a* be joined to B by a straight line. We have now to consider the motion of the point B along the circle as the chord *a* B is turning round *a* towards the position *a* T; and the difficulty in

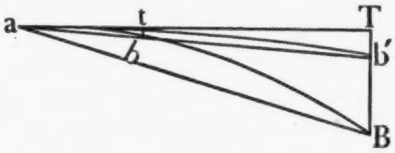


FIG. 116

our way is clearly that figures like *a* B T get small, as for example *a b t*, and continue to decrease until they cease to be large enough to be definitely observed. Newton gets over this difficulty by supposing that the figure is always magnified to a definite size; so that instead of considering the smaller figure *a b t* we magnify it throughout until *a t* is equal to the original length *a* T. But the portion *a b* of the circle with which we are now concerned is less than the former portion *a* B; consequently when it is magnified to the same length (or nearly so) it must appear straighter. That is to say, in the new figure *a b′* T, which is *a b t* magnified, the point *b′* will be nearer to the point T than B in the old one *a* B T; consequently, also, as *b* moves along to *a* the chord *a b* will get nearer to the tangent *a* T, or, what is the same thing, the angle *t a b* will get smaller. This last result is clear enough, because, as we previously supposed, the chord *a b* is always turning round towards the position *a t*.

But now the important thing is that, by taking *b* near enough to *a*, we can make the curve in the magnified figure as straight as we please; that is to say, we can make *b′* approach as near as we like to T. If we were to measure off from T perpendicularly to *a* T any length, however small, say T *d* (Fig. 117), then we can always draw a circle which shall have *a* T for a tangent and which shall pass between T and *d*; and, further, if we like to draw a line *a d* making a very small angle with *a* T, then it will still be possible to make *b* go so close to *a* that in the magnified figure the angle *b′ a* T shall be smaller than the angle *d a* T which we have drawn.

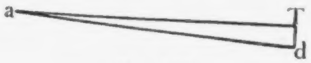


Fig. 117

Now mark what this process, which has been called Newton's microscope, really means. While the figure which we wish to study is getting smaller and smaller, and finally disappears altogether, we suppose it to be continually magnified, so as to retain a convenient size. We have one point moving along a curve up towards another point, and we want to consider what happens to the line joining them when the two points approach indefinitely near to one another. The result at which we have arrived by means of our microscope is that, by taking the points near enough together, the line may be made to approach as near as we please to the tangent to the curve at the point *a*. This, therefore, gives us a definition of the tangent to a curve in terms only of measurable quantities. If at a certain point *a* of a curve there is a line *a t* possessing the property that by taking *b* near enough to *a* on the curve the line *a b* can be brought as near as we like to *a t* (that is, the angle *b a t* made less than any assigned angle, however small), then *a t* is called the tangent to the curve at the point *a*. Observe that all the things supposed to be done in this definition are things which we know can be done. A very small angle can be assigned; then, this angle being drawn, a position of the point *b* can be found which is such that *a b* makes with *a t* an angle smaller than this. A supposition is here made in terms of quantities which we already know and can measure. We only suppose in addition that, however small the assigned angle may be, the point *b* can always be found; and if this is possible, then in the case in which the assigned angle is extremely small, the line *a b* or *a t* (for they now coincide) is called a tangent.

It is worth while to observe the likeness between this definition and the one that we previously discussed of the fourth proportional or of the equality of ratio. In that definition we supposed that, a certain fraction being assigned, if the first ratio were greater than this fraction, so also was the second ratio, and if less, less; and the question whether these ratios were greater or less is one that can be settled by measurement and comparison. We then made the further supposition that whatever fraction were assigned the same result would hold good; and we said that in that case the ratios were equal. Now in both of these definitions, applying respectively to tangents and to ratios, the difficulty is that we cause a particular supposition to be extended so as to be general; for we assume that a statement which can be very easily tested and found true in any one case is true in an infinite number of cases in which it has not been tested. But although the test cannot be applied individually to all these cases in a practical way, yet, since it is true in any individual case, we know on rational grounds that it must be satisfied in general; and therefore, justified by this knowledge, we are able to reason generally about the equality of ratios and about the tangents to curves.

Let us now translate the definition at which we have thus arrived from the language of curves and tangents into the language of instantaneous and mean velocities. The steepness of the chord of the curve of positions indicates the mean velocity, while the steepness of the tangent to the curve at any point indicates the instantaneous velocity at that point. The process of making the point *b* move nearer and nearer to the point *a* corresponds to taking for consideration a smaller and smaller interval of time after that moment at which the instantaneous velocity is wanted.

Suppose, then, the velocity of a body, viz. a railway train, to be varying, and that we want to find what its value is at a given instant. We might get a very rough approximation to it, or in some cases no approximation at all, by taking the mean velocity during the hour which follows that instant. We should get a closer approximation by taking the mean velocity during the minute succeeding that instant, because the instantaneous velocity would have less time to change. A still closer approximation would be obtained were we to take the mean velocity during the succeeding second. In all motions we should have to consider that we could make the approximation as close as we like by taking a sufficiently small interval. That is to say, if we choose to name any very small velocity, such as one with which a body going uniformly would move only an inch in a century, then, by taking the [time] interval small enough, it will be possible to make the mean velocity differ from the instantaneous velocity by less than this amount. Thus, finally, we shall have the following definition of instantaneous velocity: If there is a certain velocity to which the mean velocity during the interval succeeding a given instant can be made to approach as near as we like by taking the interval small enough, then that velocity is called the instantaneous velocity of the body at the given instant.

In this way then we have reduced the problem of finding the velocity of a moving body at any instant to the problem of drawing a tangent to its curve of positions at the corresponding point; and what we have already proved amounts to saying that, if the position of the body be given in terms of the time by means of a curve, then the velocity of the body will be given in terms of the time by means of the tangent to this curve.

Now there are many curves to which we can draw tangents by simple geometrical methods, as, for example, to the ellipse and the parabola; so that, whenever the curve of positions of a body happens to be one of these, we are able to find by geometrical construction the velocity of the body at any instant. Thus in the case of a falling body the curve of positions is a parabola, and we might find by the known properties of the tangent to a parabola that the velocity in this case is proportional to the time. But in the great majority of cases the problem of drawing a tangent to the curve of positions is just as difficult as the original problem of determining the velocity of a moving body, and in fact we do in many cases solve the former by means of the latter.[1]

§6. *On the Determination of Variable Velocity*

What is actually wanted in every case will be apparent from the consideration of the problem we have just mentioned—that of a body falling down straight. We note, from the experience of Galilei, that the whole distance which the body has fallen from rest at any instant is proportional to the square of the time; in fact, to obtain this distance in feet we must multiply the number of seconds by itself and the result by a number a little greater than sixteen. Thus, for instance, in five seconds the body will have fallen rather more than twenty-five times sixteen feet, or 400 feet. Now what we want is some direct process of proving that when the distance traversed is proportional to the square of the time the velocity is always proportional to the time. In the present case we can find the velocity at the end of a given number of seconds by multiplying that number by thirty-two feet; thus at the end of five seconds the velocity of the body will be 160 feet per second.[1] Now as a matter of fact a

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[1] The method is due to Roberval (1602–1675).

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[1] The following may be taken as a proof. Let a be the distance from rest moved over by the body in t seconds, b that moved over by it in $t + t'$ seconds, so that t' seconds is the interval we take to find out the mean velocity. Now by our rule just quoted, since a feet are passed over in t seconds, we have

$$\begin{aligned} a &= 16t^2, \\ \text{and similarly} \qquad b &= 16(t + t')^2 = 16(t^2 + 2tt' + t'^2). \\ \text{Hence we have } b - a &= 16(t^2 + 2tt' + t'^2) - 16t^2 \\ &= 16(2tt' + t'^2) \\ &= 16t'(2t + t'), \end{aligned}$$

giving the distance moved over in the interval t'. But the mean velocity during this interval is obtained by dividing the distance moved over by the time taken to traverse it; hence the mean velocity in our case for the interval of t' seconds immediately succeeding the t seconds

$$\begin{aligned} &= \frac{b - a}{t'} \\ &= \frac{16t'(2t + t')}{t'} \\ &= 16(2t + t') \\ &= 32t + 16t'. \end{aligned}$$

Now if we look at this result, which we have obtained for the mean velocity, we see that there are two terms in it. The first, viz. $32t$, is quite independent of the interval t' which we have taken; the second, viz. $16t'$, depends directly on it, and will therefore change when we change the interval. Now the distance per second represented by $16t'$ feet can be made as small as we like by taking t' small enough; so that the mean velocity during the interval t' seconds succeeding the given instant can be made to approach $32t$ feet per second as near as we like by taking t' small enough. Recurring to our definition of instantaneous velocity, it is now evident that the instantaneous velocity of our falling body at the end of t seconds is $32t$ feet per second.

process (of which there is a simple example in the footnote) has been worked out, by which from any algebraical rule telling us how to calculate the distance traversed in terms of the time we can find another algebraical rule which will tell us how to calculate the velocity in terms of the time. One case of the process is this: If the distance traversed is at any instant a times the nth power of the time, then the velocity at any instant will be na times the $(n-1)$th power of the time. It is by means of this process of altering one algebraical rule so as to get another from it that both of the problems which we have shown to be equivalent to one another are solved in practice.

There is yet another problem of very great importance in the study of natural phenomena which can be made to depend on these two. When a point moves along a straight line the distance of it from some fixed point in the line is a quantity which varies from time to time. The rate of change of this distance is the same thing as the velocity of the moving point; and the rate of change of any continuous quantity can only be properly represented by means of the velocity of a point.

Thus, for instance, the height of the tide at a given port will vary from time to time during the day, and it may be indicated by a mark which goes up and down on a stick. The rate at which the height of the tide varies will obviously be the same thing as the velocity with which this mark goes up and down. Again the pressure of the atmosphere is indicated by means of the height of a mercury barometer. The rate at which this pressure changes is obviously the same thing as the velocity with which the surface of the mercury moves up and down. Now whenever we want to describe the changes which take place in any quantity in terms of the time, we may indeed roughly and approximately do so by means of a table. But this is also the most troublesome way; the proper way of describing them is by drawing a curve in which the *abscissa*, or horizontal distance, at any point represents the time, while the height of the curve at that point represents the value of the quantity at that time (see p. 167). Whenever this is done we practically suppose the variation of the quantity to be represented by the motion of the point on a curve. The quantity can only be adequately represented by marking off a length proportional to it on a line; so that if the quantity varies then the length marked off will vary, and consequently the end of this length will move along the curve. The rate at which the quantity varies is the rate at which this point moves; and when the values of the quantity for different times are represented by the perpendicular distances of points on a curve from the line which represents the time, its rate of variation is determined by the tangent to that curve.

§7. *On the Method of Fluxions*

Hence we have three problems which are practically the same. First, to find the velocity of a moving point when we know where it is at every instant; secondly, to draw a tangent to a curve at any point; thirdly, to find the rate of change of a quantity when we know how great it is at every instant. And the solution of them all depends upon that process by which, when we take the algebraical rule for finding the quantity in terms of the time, we deduce from it another rule for finding its rate of change in terms of the time.

This particular process of deriving one algebraical rule from another was first investigated by Newton. He was accustomed to describe a varying quantity as a *fluent*, and its rate of change he called the *fluxion* of the quantity. On account of these names, the entire method of solving these problems by means of the process of deriving one algebraical rule from another was termed the *Method of Fluxions*.

In general the rate of variation of a quantity will itself change from time to time; but if we consider only an interval very small as compared with that required for a considerable variation of the quantity, we may legitimately suppose that it has not altered much during that interval. This is practically equivalent to supposing that the law of change has been uniformly true during that interval, and that the rate of change does not differ very much from its mean value. Now the mean rate of change of a quantity during an interval of time is just the difference between the values of the quantity at the beginning and at the end divided by the interval. If any quantity increased by one inch in a second, then, although it may not have been increasing uniformly, or even been increasing at all during the whole of that second, yet during the second its mean rate of increase was one inch per second. Now if the rate of increase only changes slowly we may, as an approximation, fairly suppose it to be constant during the second, and therefore to be equal to the mean rate; and, as we know, the smaller the interval of time is, the less is the error arising from this supposition. This is, as a matter of fact, the way in which that process is established by means of which a rule for calculating position is altered into a rule for calculating velocity. The difference between the distances of the moving point from some fixed point on the line at two different times is divided by the interval between the times, and this gives the mean rate of change during that interval. If we find that, by making the interval smaller and smaller, this mean rate of change gets nearer and nearer to a certain value, then we conclude that this value is the actual rate of change when we suppose the interval to shrink up into an instant, or that it is, as we call it, the instantaneous rate of change.

Because two differences are used in the argument which establishes the process for changing the one rule into the other, this process was called, first in other countries and then also in England, the *Differential Calculus*. The name is an unfortunate one, because the rate of change which is therein calculated has nothing to do with differences, the only connection with differences being that they are mentioned in the argument which is used to establish the process. However this may be, the object of the differential calculus or of the method of fluxions (whichever name we choose to give it) is to find a rule for calculating the rate of change of a quantity when we have a rule for calculating the quantity itself; and we have seen that when this can be done the problem of drawing a tangent to a curve and that of finding the velocity of a moving point are also solved.

§8. *Of the Relationship of Quantities, or Functions*

But we not only have rules for calculating the value of a quantity at any time, but also rules for calculating the value of one quantity in terms of another quite independently of the time. Of the former class of rules an example is the one mentioned above for calculating the rise of the tide. We may either write down a formula which will enable us to calculate it at a given instant, or we may draw a curve which shall represent its rise at different times of the day. Of the second kind of rule a good example is that in which the pressure of a given quantity of gas is given in terms of its volume when the temperature is supposed to be constant; the algebraical statement of the rule giving the relation between them is that the two things vary inversely as one another, or that the product representing them is constant.

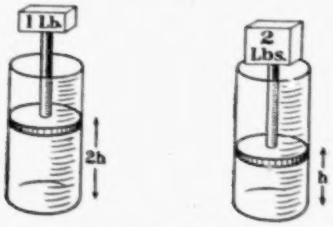

1 Lb.
2h
2 Lbs.
h

FIG. 118

Thus if we compress a mass of air to one-half of its natural volume the pressure will become twice as great, or will be, as it is called, two "atmospheres." And so if we compress it to one-fifth of the volume the pressure will become five times as great, or five atmospheres (Fig. 118).

Galileo uses a thought experiment in discussing projectile motion, a typical device of the scientist to this day. Galileo's book was originally published in 1632.

11 Galileo's Discussion of Projectile Motion

Gerald Holton and Duane H. D. Roller

1958

3.1 Galileo's discussion of projectile motion. To this point we have been solely concerned with the motion of objects as characterized by their speed; we have not given much consideration to the *direction* of motion, or to changes in direction of motion. Turning now to the more general problem of projectile motion, we leave the relatively simple case of bodies moving in a *straight line* only and expand our methods to deal with projectiles moving along curved paths. Our understanding of this field will hinge largely on a far-reaching idea: the observed motion of a projectile may be thought of as the result of two *separate* motions, combined and occurring *simultaneously;* one component of motion is in a horizontal direction and without acceleration, whereas the other is in a vertical direction and has a constant acceleration downward in accordance with the laws of free fall. Furthermore, these two components do not interfere with each other; each component may be studied as if the other were not present. Thus the whole motion of the projectile at every moment is simply the result of the two individual actions.

This principle of the independency of the horizontal and vertical components of projectile motion was set forth by Galileo in his *Dialogue on the great world systems* (1632). Although in this work he was principally concerned with astronomy, Galileo already knew that terrestrial mechanics offered the clue to a better understanding of planetary motions. Like the *Two new sciences*, this earlier work is cast in the form of a discussion among the same three characters, and also uses the Socratic method of the Platonic dialogues. Indeed, the portion of interest to us here begins with Salviati reiterating one of Socrates' most famous phrases, as he tells the Aristotelian Simplicio that he, Simplicio, knows far more about mechanics than he is aware:*

> *Salviati:* . . . Yet I am so good a midwife of minds that I will make you confess the same whether you will or no. But Sagredus stands very quiet, and yet, if I mistake not, I saw him make some move as if to speak.
>
> *Sagredo:* I had intended to speak a fleeting something; but my curiosity aroused by your promising that you would force Simplicius to uncover the knowledge which he conceals from us has made me depose all other thoughts. Therefore I pray you to make good your vaunt.

---

*These extracts from Galileo's *Dialogue on the great world systems*, as well as those appearing in later chapters, are taken from the translation of T. Salusbury, edited and corrected by Giorgio de Santillana (University of Chicago Press, 1953).

> *Salviati:* Provided that Simplicius consents to reply to what I shall ask him, I will not fail to do it.
>
> *Simplicio:* I will answer what I know, assured that I shall not be much put to it, for, of those things which I hold to be false, I think nothing can be known, since Science concerns truths, not falsehoods.
>
> *Salviati:* I do not desire that you should say that you know anything, save that which you most assuredly know. Therefore, tell me; if you had here a flat surface as polished as a mirror and of a substance as hard as steel that was not horizontal but somewhat inclining, and you put upon it a perfectly spherical ball, say, of bronze, what do you think it would do when released? Do you not believe (as for my part I do) that it would lie still?
>
> *Simplicio:* If the surface were inclining?
>
> *Salviati:* Yes, as I have already stated.
>
> *Simplicio:* I cannot conceive how it should lie still. I am confident that it would move towards the declivity with much propenseness.
>
> *Salviati:* Take good heed what you say, Simplicius, for I am confident that it would lie still in whatever place you should lay it.
>
> *Simplicio:* So long as you make use of such suppositions, Salviatus, I shall cease to wonder if you conclude most absurd conclusions.
>
> *Salviati:* Are you assured, then, that it would freely move towards the declivity?
>
> *Simplicio:* Who doubts it?
>
> *Salviati:* And this you verily believe, not because I told you so (for I endeavored to persuade you to think the contrary), but of yourself, and upon your natural judgment?
>
> *Simplicio:* Now I see your game; you did not say this really believing it, but to try me, and to wrest words out of my mouth with which to condemn me.
>
> *Salviati:* You are right. And how long and with what velocity would that ball move? But take notice that I gave as the example a ball exactly round, and a plane exquisitely polished, so that all external and accidental impediments might be taken away. Also I would have you remove all obstructions caused by the air's resistance and any other causal obstacles, if any other there can be.
>
> *Simplicio:* I understand your meaning very well and answer that the ball would continue to move *in infinitum* if the inclination of the plane should last so long, accelerating continually. Such is the nature of ponderous bodies that they acquire strength in going, and, the greater the declivity, the greater the velocity will be.

Simplicio is next led to express his belief that if he observed the ball rolling *up* the inclined plane he would know that it had been pushed or thrown, since it is moving contrary to its natural tendencies. Then Salviati turns to the intermediate case:

> *Salviati:* It seems, then, that hitherto you have well explained to me the accidents of a body on two different planes. Now tell me, what would befall the same body upon a surface that had neither acclivity nor declivity?

Simplicio: Here you must give me a little time to consider my answer. There being no declivity, there can be no natural inclination to motion; and there being no acclivity, there can be no resistance to being moved. There would then arise an indifference between propulsion and resistance; therefore, I think it ought naturally stand still. But I had forgot myself; it was not long ago that Sagredus gave me to understand that it would do so.

Salviati: So I think, provided one did lay it down gently; but, if it had an impetus directing it towards any part, what would follow?

Simplicio: That it should move towards that part.

Salviati: But with what kind of motion? Continually accelerated, as in declining planes; or successively retarded, as in those ascending?

Simplicio: I cannot tell how to discover any cause of acceleration or retardation, there being no declivity or acclivity.

Salviati: Well, if there be no cause of retardation, even less should there be any cause of rest. How long therefore would you have the body move?

Simplicio: As long as that surface, neither inclined nor declined, shall last.

Salviati: Therefore if such a space were interminate, the motion upon it would likewise have no termination, that is, would be perpetual.

Simplicio: I think so, if the body is of a durable matter.

Salviati: That has been already supposed when it was said that all external and accidental impediments were removed, and the brittleness of the body in this case is one of those accidental impediments. Tell me now, what do you think is the cause that that same ball moves spontaneously upon the inclining plane, and does not, except with violence, upon the plane sloping upwards?

Simplicio: Because the tendency of heavy bodies is to move towards the center of the Earth and only by violence upwards towards the circumference. [This is the kernel of the Scholastic viewpoint on falling bodies (see Section 2.3). Salviati does not refute it, but turns it to Galileo's purposes.]

Salviati: Therefore a surface which should be neither declining nor ascending ought in all its parts to be equally distant from the center. But is there any such surface in the world?

Simplicio: There is no want of it, such is our terrestrial globe, for example, if it were not rough and mountainous. But you have that of the water, at such time as it is calm and still.

Here is the genesis of one of the fundamental principles of the new mechanics: if all "accidental" interferences with an object's motion are removed, the motion will endure. The "accidents" are eliminated in this thought experiment by: (1) proposing the use of a perfectly round, perfectly hard ball on a perfectly smooth surface, and (2) by imagining the surface to be a globe whose surface is everywhere equidistant from the center of the earth, so that the ball's "natural tendency" to go downward is balanced by the upward thrust of the surface. (We shall return to this latter point in our discussion of isolated systems in Chapter 16.) Note carefully the drastic change from the Scholastic view: instead of asking "What makes the ball move?" Galileo asks "What might change its motion?"

Having turned the conversation to smooth water, Galileo brings in the motion of a stone dropping from the mast of a moving ship. Since the stone is moving horizontally with the ship before it is dropped, it should continue to move horizontally while it falls.

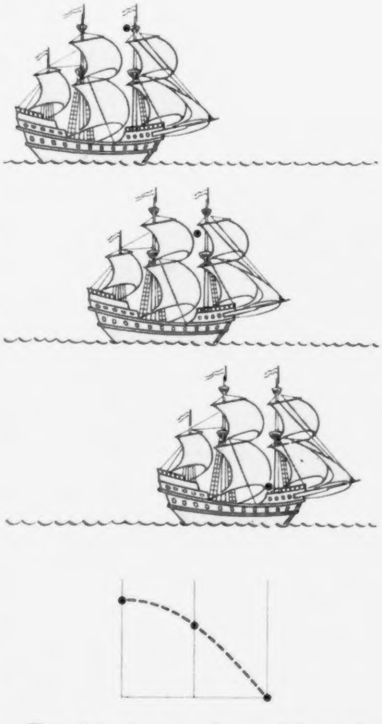


Fig. 3.1. A stone dropped from the mast of a ship in uniform motion. From the shore the trajectory of the stone is seen to be a curved line (parabola).

Sagredo: If it be true that the impetus with which the ship moves remains indelibly impressed in the stone after it is let fall from the mast; and if it be further true that this motion brings no impediment or retardment to the motion directly downwards natural to the stone, then there ought to ensue an effect of a very wonderful nature. Suppose a ship stands still, and the time of the falling of a stone from the mast's round top to the deck is two beats of the pulse. Then afterwards have the ship under sail and let the same stone depart from the same place. According to what has been premised, it shall still take up the time of two pulses in its fall, in which time the ship will have gone, say, twenty yards. The true motion of the stone then will be a transverse line [i.e., a curved line in the vertical plane, see Fig. 3.1], considerably longer than the first straight and perpendicular line, the height of the mast, and yet nevertheless the stone will have passed it in the same time. Increase the ship's velocity as much as you will, the falling stone shall describe its transverse lines still longer and longer and yet shall pass them all in those selfsame two pulses. In this same fashion, if a cannon were leveled on the top of a tower, and fired point-blank, that is, horizontally, and whether the charge were small or large with the ball falling sometimes a thousand yards distant, sometimes four thousand, sometimes ten, etc., all these shots shall come to ground in times equal to each other. And every one equal to the time that the ball would take to pass from the mouth of the piece to the ground, if, without other impulse, it falls simply downwards in a perpendicular line. Now it seems a very admirable thing that, in the same short time of its falling perpendicularly down to the ground from the height of, say, a hundred yards, equal balls, fired violently out of the piece,

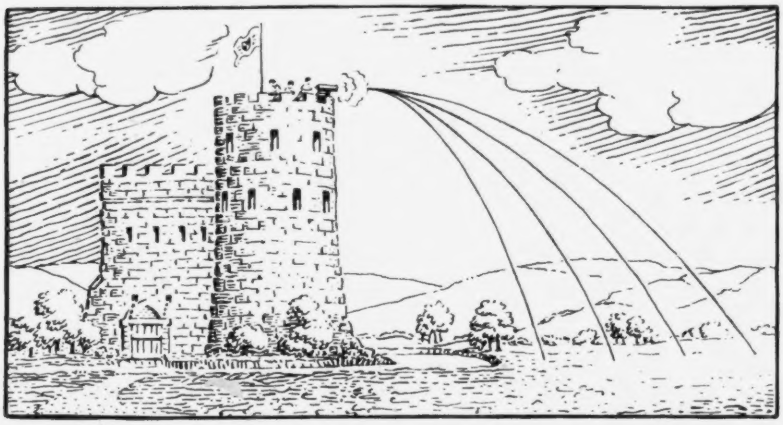


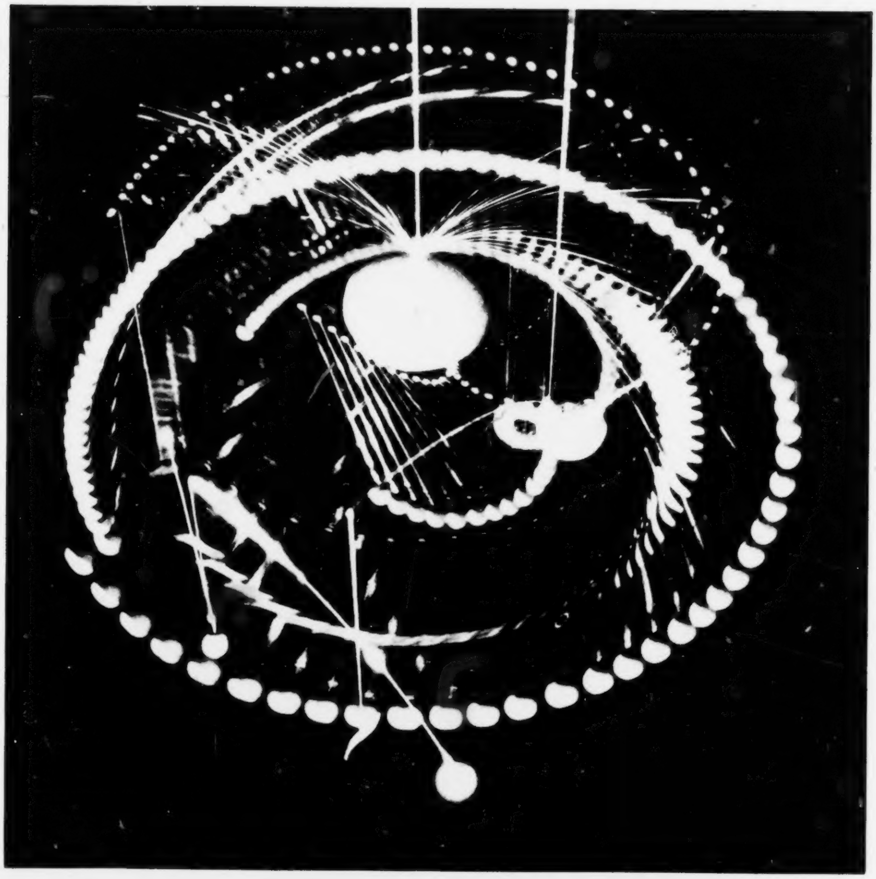
FIG. 3.2. For cannon balls fired horizontally with different initial forward speeds, "all the balls in all the shots made horizontally remain in the air an equal time."

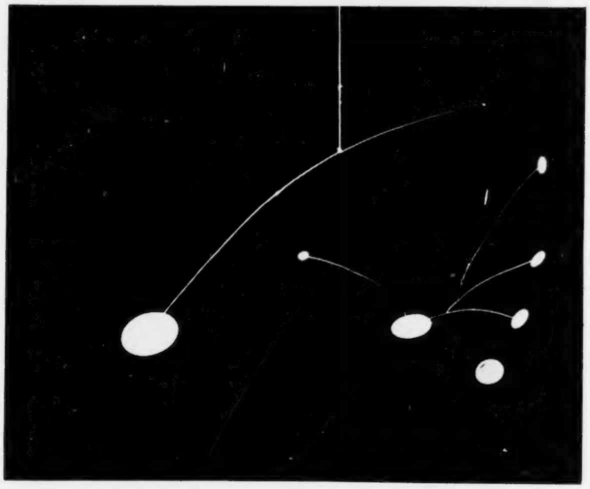
should be able to pass four hundred, a thousand, even ten thousand yards. All the balls in all the shots made horizontally remain in the air an equal time [Fig. 3.2].

Salviati: The consideration is very elegant for its novelty and, if the effect be true, very admirable. Of its truth I make no question, and, were it not for the accidental impediment of the air, I verily believe that, if at the time of the ball's going out of the piece another were let fall from the same height directly downwards, they would both come to the ground at the same instant, though one should have traveled ten thousand yards in its range, and another only a hundred, presupposing the surface of the Earth to be level. As for the impediment which might come from the air, it would consist in retarding the extreme swift motion of the shot.

3.2 Projectile launched horizontally. Galileo's two thought experiments may be rephrased and analyzed in terms of two modern examples.

(1) If we watch an airplane in steady horizontal flight drop a small, heavy object, we shall see that the object remains very nearly *directly below* the plane while, of course, dropping closer and closer to the ground (Fig. 3.3). If we had been riding in the plane when the object was dropped, we would have seen only the horizontal part (component) of the motion; that is, we would have seen the object traveling along directly under the plane, although of course appearing to become smaller and smaller as it receded toward the ground. The clear implication is that *the horizontal component of the motion of the object remains what it was at the moment of release from the plane,* even though there is superposed on it the ever-increasing speed downward.





This chapter from a beginning college physics text is not simple, but the reward of this numerical approach to Newtonian mechanics is a more powerful understanding of how the laws of motion work.

12

Newton's Laws of Dynamics

Richard P. Feynman, Robert B. Leighton and Matthew Sands

1963

9-1 Momentum and force

The discovery of the laws of dynamics, or the laws of motion, was a dramatic moment in the history of science. Before Newton's time, the motions of things like the planets were a mystery, but after Newton there was complete understanding. Even the slight deviations from Kepler's laws, due to the perturbations of the planets, were computable. The motions of pendulums, oscillators with springs and weights in them, and so on, could all be analyzed completely after Newton's laws were enunciated. So it is with this chapter: before this chapter we could not calculate how a mass on a spring would move; much less could we calculate the perturbations on the planet Uranus due to Jupiter and Saturn. After this chapter we *will* be able to compute not only the motion of the oscillating mass, but also the perturbations on the planet Uranus produced by Jupiter and Saturn!

Galileo made a great advance in the understanding of motion when he discovered the *principle of inertia:* if an object is left alone, is not disturbed, it continues to move with a constant velocity in a straight line if it was originally moving, or it continues to stand still if it was just standing still. Of course this never appears to be the case in nature, for if we slide a block across a table it stops, but that is because it is *not* left to itself—it is rubbing against the table. It required a certain imagination to find the right rule, and that imagination was supplied by Galileo.

Of course, the next thing which is needed is a rule for finding how an object *changes* its speed if something *is* affecting it. That is the contribution of Newton. Newton wrote down three laws: The First Law was a mere restatement of the Galilean principle of inertia just described. The Second Law gave a specific way of determining how the velocity changes under different influences called *forces*. The Third Law describes the forces to some extent, and we shall discuss that at another time. Here we shall discuss only the Second Law, which asserts that the motion of an object is changed by forces in this way: *the time-rate-of-change of a quantity called momentum is proportional to the force.* We shall state this mathematically shortly, but let us first explain the idea.

Momentum is not the same as *velocity*. A lot of words are used in physics, and they all have precise meanings in physics, although they may not have such precise meanings in everyday language. Momentum is an example, and we must define it precisely. If we exert a certain push with our arms on an object that is light, it moves easily; if we push just as hard on another object that is much heavier in the usual sense, then it moves much less rapidly. Actually, we must change the words from "light" and "heavy" to *less massive* and *more massive*, because there is a difference to be understood between the *weight* of an object and its *inertia.* (How hard it is to get it going is one thing, and how much it weighs is something else.) Weight and inertia are *proportional*, and on the earth's surface are often taken to be numerically equal, which causes a certain confusion to the student. On Mars, weights would be different but the amount of force needed to overcome inertia would be the same.

We use the term *mass* as a quantitative measure of inertia, and we may measure mass, for example, by swinging an object in a circle at a certain speed and measuring how much force we need to keep it in the circle. In this way we find a certain quantity of mass for every object. Now the *momentum* of an object is a product of two parts: its *mass* and its *velocity*. Thus Newton's Second Law may be written mathematically this way:

$$F = \frac{d}{dt}(mv). \tag{9.1}$$

Now there are several points to be considered. In writing down any law such as this, we use many intuitive ideas, implications, and assumptions which are at first combined approximately into our "law." Later we may have to come back and study in greater detail exactly what each term means, but if we try to do this too soon we shall get confused. Thus at the beginning we take several things for granted. First, that the mass of an object is *constant*; it isn't really, but we shall start out with the Newtonian approximation that mass is constant, the same all the time, and that, further, when we put two objects together, their masses *add.* These ideas were of course implied by Newton when he wrote his equation, for otherwise it is meaningless. For example, suppose the mass varied inversely as the velocity; then the momentum would *never change* in any circumstance, so the law means nothing unless you know how the mass changes with velocity. At first we say, *it does not change.*

Then there are some implications concerning force. As a rough approximation we think of force as a kind of push or pull that we make with our muscles, but we can define it more accurately now that we have this law of motion. The most important thing to realize is that this relationship involves not only changes in the *magnitude* of the momentum or of the velocity but also in their *direction.*

If the mass is constant, then Eq. (9.1) can also be written as

$$F = m\frac{dv}{dt} = ma. \tag{9.2}$$

The acceleration a is the rate of change of the velocity, and Newton's Second Law says more than that the effect of a given force varies inversely as the mass; it says also that the *direction* of the change in the velocity and the *direction* of the force are the same. Thus we must understand that a change in a velocity, or an acceleration, has a wider meaning than in common language: The velocity of a moving object can change by its speeding up, slowing down (when it slows down, we say it accelerates with a negative acceleration), or changing its direction of motion. An acceleration at right angles to the velocity was discussed in Chapter 7. There we saw that an object moving in a circle of radius R with a certain speed v along the circle falls away from a straightline path by a distance equal to $\frac{1}{2}(v^2/R)t^2$ if t is very small. Thus the formula for acceleration at right angles to the motion is

$$a = v^2/R, \tag{9.3}$$

and a force at right angles to the velocity will cause an object to move in a curved path whose radius of curvature can be found by dividing the force by the mass to get the acceleration, and then using (9.3).

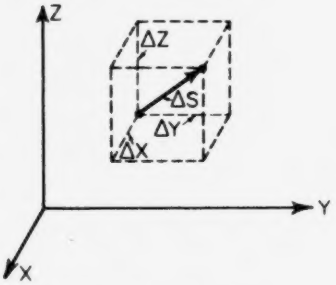


Fig. 9–1. A small displacement of an object.

9–2 Speed and velocity

In order to make our language more precise, we shall make one further definition in our use of the words *speed* and *velocity*. Ordinarily we think of speed and velocity as being the same, and in ordinary language they are the same. But in physics we have taken advantage of the fact that there *are* two words and have chosen to use them to distinguish two ideas. We carefully distinguish velocity, which has both magnitude and direction, from speed, which we choose to mean the magnitude of the velocity, but which does not include the direction. We can formulate this more precisely by describing how the x-, y-, and z-coordinates of an object change with time. Suppose, for example, that at a certain instant an object is moving as shown in Fig. 9–1. In a given small interval of time Δt it will move a certain distance Δx in the x-direction, Δy in the y-direction, and Δz in the z-direction. The total effect of these three coordinate changes is a displacement Δs along the diagonal of a parallelepiped whose sides are Δx, Δy, and Δz. In terms of the velocity, the displacement Δx is the x-component of the velocity times Δt, and similarly for Δy and Δz:

$$\Delta x = v_x\,\Delta t, \qquad \Delta y = v_y\,\Delta t, \qquad \Delta z = v_z\,\Delta t. \tag{9.4}$$

9-3 Components of velocity, acceleration, and force

In Eq. (9.4) *we have resolved the velocity into components* by telling how fast the object is moving in the x-direction, the y-direction, and the z-direction. The velocity is completely specified, both as to magnitude and direction, if we give the numerical values of its three rectangular components:

$$v_x = dx/dt, \qquad v_y = dy/dt, \qquad v_z = dz/dt. \tag{9.5}$$

On the other hand, the speed of the object is

$$ds/dt = |v| = \sqrt{v_x^2 + v_y^2 + v_z^2}. \tag{9.6}$$

Next, suppose that, because of the action of a force, the velocity changes to some other direction and a different magnitude, as shown in Fig. 9-2. We can analyze this apparently complex situation rather simply if we evaluate the changes in the x-, y-, and z-components of velocity. The change in the component of the velocity in the x-direction in a time Δt is $\Delta v_x = a_x\,\Delta t$, where a_x is what we call the x-component of the acceleration. Similarly, we see that $\Delta v_y = a_y\,\Delta t$ and $\Delta v_z = a_z\,\Delta t$. In these terms, we see that Newton's Second Law, in saying that the force is in the same direction as the acceleration, is really three laws, in the sense that the component of the force in the x-, y-, or z-direction is equal to the mass times

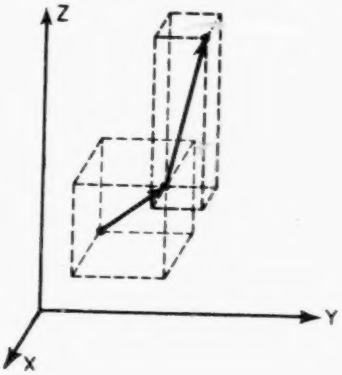


Fig. 9-2. A change in velocity in which both the magnitude and direction change.

the rate of change of the corresponding component of velocity:

$$\begin{aligned} F_x &= m(dv_x/dt) = m(d^2x/dt^2) = ma_x, \\ F_y &= m(dv_y/dt) = m(d^2y/dt^2) = ma_y, \\ F_z &= m(dv_z/dt) = m(d^2z/dt^2) = ma_z. \end{aligned} \tag{9.7}$$

Just as the velocity and acceleration have been resolved into components by projecting a line segment representing the quantity and its direction onto three coordinate axes, so, in the same way, a force in a given direction is represented by certain components in the x-, y-, and z-directions:

$$\begin{aligned} F_x &= F \cos(x, F), \\ F_y &= F \cos(y, F), \\ F_z &= F \cos(z, F), \end{aligned} \tag{9.8}$$

where F is the magnitude of the force and (x, F) represents the angle between the x-axis and the direction of F, etc.

Newton's Second Law is given in complete form in Eq. (9.7). If we know the forces on an object and resolve them into x-, y-, and z-components, then we can find the motion of the object from these equations. Let us consider a simple example. Suppose there are no forces in the y- and z-directions, the only force being in the x-direction, say vertically. Equation (9.7) tells us that there would be changes in the velocity in the vertical direction, but no changes in the horizontal direction. This was demonstrated with a special apparatus in Chapter 7 (see Fig. 7–3). A falling body moves horizontally without any change in horizontal motion, while it moves vertically the same way as it would move if the horizontal motion were zero. In other words, motions in the x-, y-, and z-directions are independent if the *forces* are not connected.

9–4 What is the force?

In order to use Newton's laws, we have to have some formula for the force; these laws say *pay attention to the forces.* If an object is accelerating, some agency is at work; find it. Our program for the future of dynamics must be to *find the laws for the force.* Newton himself went on to give some examples. In the case of gravity he gave a specific formula for the force. In the case of other forces he gave some part of the information in his Third Law, which we will study in the next chapter, having to do with the equality of action and reaction.

Extending our previous example, what are the forces on objects near the earth's surface? Near the earth's surface, the force in the vertical direction due to gravity is proportional to the mass of the object and is nearly independent of height for heights small compared with the earth's radius R: $F = GmM/R^2 = mg$, where $g = GM/R^2$ is called the *acceleration of gravity.* Thus the law of gravity tells us that weight is proportional to mass; the force is in the vertical direction and is the mass times g. Again we find that the motion in the horizontal direction

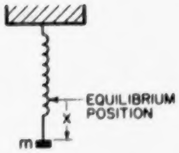


Fig. 9–3. A mass on a spring.

is at constant velocity. The interesting motion is in the vertical direction, and Newton's Second Law tells us

$$mg = m(d^2x/dt^2). \tag{9.9}$$

Cancelling the m's, we find that the acceleration in the x-direction is constant and equal to g. This is of course the well known law of free fall under gravity, which leads to the equations

$$\begin{aligned} v_x &= v_0 + gt, \\ x &= x_0 + v_0t + \tfrac{1}{2}gt^2. \end{aligned} \tag{9.10}$$

As another example, let us suppose that we have been able to build a gadget (Fig. 9–3) which applies a force proportional to the distance and directed oppositely —a spring. If we forget about gravity, which is of course balanced out by the initial stretch of the spring, and talk only about *excess* forces, we see that if we pull the mass down, the spring pulls up, while if we push it up the spring pulls down. This machine has been designed carefully so that the force is greater, the more we pull it up, in exact proportion to the displacement from the balanced condition, and the force upward is similarly proportional to how far we pull down. If we watch the dynamics of this machine, we see a rather beautiful motion—up, down, up, down, . . . The question is, will Newton's equations correctly describe this motion? Let us see whether we can exactly calculate how it moves with this periodic oscillation, by applying Newton's law (9.7). In the present instance, the equation is

$$-kx = m(dv_x/dt). \tag{9.11}$$

Here we have a situation where the velocity in the x-direction changes at a rate proportional to x. Nothing will be gained by retaining numerous constants, so we shall imagine either that the scale of time has changed or that there is an accident in the units, so that we happen to have $k/m = 1$. Thus we shall try to solve the equation

$$dv_x/dt = -x. \tag{9.12}$$

To proceed, we must know what v_x is, but of course we know that the velocity is the rate of change of the position.

9–5 Meaning of the dynamical equations

Now let us try to analyze just what Eq. (9.12) means. Suppose that at a given time t the object has a certain velocity v_x and position x. What is the velocity and what is the position at a slightly later time $t + \epsilon$? If we can answer this question our problem is solved, for then we can start with the given condition and compute how it changes for the first instant, the next instant, the next instant, and so on, and in this way we gradually evolve the motion. To be specific, let us suppose that at the time $t = 0$ we are given that $x = 1$ and $v_x = 0$. Why does the object move at all? Because there is a *force* on it when it is at any position except $x = 0$. If $x > 0$, that force is upward. Therefore the velocity which is zero starts to change, because of the law of motion. Once it starts to build up some velocity the object starts to move up, and so on. Now at any time t, if ϵ is very small, we may express the position at time $t + \epsilon$ in terms of the position at time t and the velocity at time t to a very good approximation as

$$x(t + \epsilon) = x(t) + \epsilon v_x(t). \tag{9.13}$$

The smaller the ϵ, the more accurate this expression is, but it is still usefully accurate even if ϵ is not vanishingly small. Now what about the velocity? In order to get the velocity later, the velocity at the time $t + \epsilon$, we need to know how the velocity changes, the *acceleration*. And how are we going to find the acceleration? That is where the law of dynamics comes in. The law of dynamics tells us what the acceleration is. It says the acceleration is $-x$.

$$\begin{aligned} v_x(t + \epsilon) &= v_x(t) + \epsilon a_x(t) && (9.14)\\ &= v_x(t) - \epsilon x(t). && (9.15)\end{aligned}$$

Equation (9.14) is merely kinematics; it says that a velocity changes because of the presence of acceleration. But Eq. (9.15) is *dynamics*, because it relates the acceleration to the force; it says that at this particular time for this particular problem, you can replace the acceleration by $-x(t)$. Therefore, if we know both the x and v at a given time, we know the acceleration, which tells us the new velocity, and we know the new position—this is how the machinery works. The velocity changes a little bit because of the force, and the position changes a little bit because of the velocity.

9-6 Numerical solution of the equations

Now let us really solve the problem. Suppose that we take $\epsilon = 0.100$ sec. After we do all the work if we find that this is not small enough we may have to go back and do it again with $\epsilon = 0.010$ sec. Starting with our initial value $x(0) = 1.00$, what is $x(0.1)$? It is the old position $x(0)$ plus the velocity (which is zero) times 0.10 sec. Thus $x(0.1)$ is still 1.00 because it has not yet started to move. But the new velocity at 0.10 sec will be the old velocity $v(0) = 0$ plus ϵ times the acceleration. The acceleration is $-x(0) = -1.00$. Thus

$$v(0.1) = 0.00 - 0.10 \times 1.00 = -0.10.$$

Now at 0.20 sec

$$x(0.2) = x(0.1) + \epsilon v(0.1)$$
$$= 1.00 - 0.10 \times 0.10 = 0.99$$

and

$$v(0.2) = v(0.1) + \epsilon a(0.1)$$
$$= -0.10 - 0.10 \times 1.00 = -0.20.$$

And so, on and on and on, we can calculate the rest of the motion, and that is just what we shall do. However, for practical purposes there are some little tricks by which we can increase the accuracy. If we continued this calculation as we have started it, we would find the motion only rather crudely because $\epsilon = 0.100$ sec is rather crude, and we would have to go to a very small interval, say $\epsilon = 0.01$. Then to go through a reasonable total time interval would take a lot of cycles of computation. So we shall organize the work in a way that will increase the precision of our calculations, using the same coarse interval $\epsilon = 0.10$ sec. This can be done if we make a subtle improvement in the technique of the analysis.

Notice that the new position is the old position plus the time interval ϵ times the velocity. But the velocity *when?* The velocity at the beginning of the time interval is one velocity and the velocity at the end of the time interval is another velocity. Our improvement is to use the velocity *halfway between.* If we know the speed now, but the speed is changing, then we are not going to get the right answer by going at the same speed as now. We should use some speed between the "now" speed and the "then" speed at the end of the interval. The same considerations also apply to the velocity: to compute the velocity changes, we should use the acceleration midway between the two times at which the velocity is to be found. Thus the equations that we shall actually use will be something like this: the position later is equal to the position before plus ϵ times the velocity *at the time in the middle of the interval.* Similarly, the velocity at this halfway point is the velocity at a time ϵ before (which is in the middle of the previous interval) plus ϵ times the acceleration at the time t. That is, we use the equations

$$\begin{aligned} x(t+\epsilon) &= x(t) + \epsilon v(t+\epsilon/2), \\ v(t+\epsilon/2) &= v(t-\epsilon/2) + \epsilon a(t), \\ a(t) &= -x(t). \end{aligned} \tag{9.16}$$

There remains only one slight problem: what is $v(\epsilon/2)$? At the start, we are given $v(0)$, not $v(-\epsilon/2)$. To get our calculation started, we shall use a special equation, namely, $v(\epsilon/2) = v(0) + (\epsilon/2)a(0)$.

Now we are ready to carry through our calculation. For convenience, we may arrange the work in the form of a table, with columns for the time, the position, the velocity, and the acceleration, and the in-between lines for the velocity, as shown in Table 9–1. Such a table is, of course, just a convenient way of representing the numerical values obtained from the set of equations (9.16), and in fact the equations themselves need never be written. We just fill in the various spaces in

Table 9–1

Solution of $dv_x/dt = -x$

Interval: $\epsilon = 0.10$ sec

t	x	v_x	a_x
0.0	1.000	0.000	−1.000
		−0.050	
0.1	0.995		−0.995
		−0.150	
0.2	0.980		−0.980
		−0.248	
0.3	0.955		−0.955
		−0.343	
0.4	0.921		−0.921
		−0.435	
0.5	0.877		−0.877
		−0.523	
0.6	0.825		−0.825
		−0.605	
0.7	0.764		−0.764
		−0.682	
0.8	0.696		−0.696
		−0.751	
0.9	0.621		−0.621
		−0.814	
1.0	0.540		−0.540
		−0.868	
1.1	0.453		−0.453
		−0.913	
1.2	0.362		−0.362
		−0.949	
1.3	0.267		−0.267
		−0.976	
1.4	0.169		−0.169
		−0.993	
1.5	0.070		−0.070
		−1.000	
1.6	−0.030		+0.030

the table one by one. This table now gives us a very good idea of the motion: it starts from rest, first picks up a little upward (negative) velocity and it loses some of its distance. The acceleration is then a little bit less but it is still gaining speed. But as it goes on it gains speed more and more slowly, until as it passes $x = 0$ at about $t = 1.50$ sec we can confidently predict that it will keep going, but now it will be on the other side; the position x will become negative, the ac-

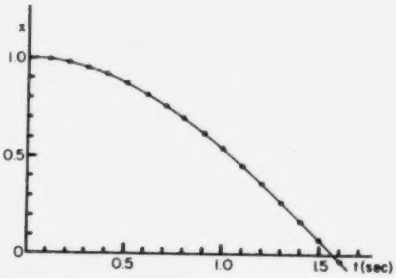


Fig. 9–4. Graph of the motion of a mass on a spring.

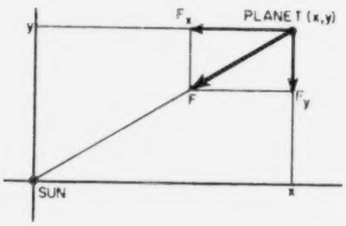


Fig. 9–5. The force of gravity on a planet.

celeration therefore positive. Thus the speed decreases. It is interesting to compare these numbers with the function $x = \cos t$, which is done in Fig. 9–4. The agreement is within the three significant figure accuracy of our calculation! We shall see later that $x = \cos t$ is the exact mathematical solution of our equation of motion, but it is an impressive illustration of the power of numerical analysis that such an easy calculation should give such precise results.

9–7 Planetary motions

The above analysis is very nice for the motion of an oscillating spring, but can we analyze the motion of a planet around the sun? Let us see whether we can arrive at an approximation to an ellipse for the orbit. We shall suppose that the sun is infinitely heavy, in the sense that we shall not include its motion. Suppose a planet starts at a certain place and is moving with a certain velocity; it goes around the sun in some curve, and we shall try to analyze, by Newton's laws of motion and his law of gravitation, what the curve is. How? At a given moment it is at some position in space. If the radial distance from the sun to this position is called r, then we know that there is a force directed inward which, according to the law of gravity, is equal to a constant times the product of the sun's mass and the planet's mass divided by the square of the distance. To analyze this further we must find out what acceleration will be produced by this force. We shall need the *components* of the acceleration along two directions, which we call x and y. Thus if we specify the position of the planet at a given moment by giving x and y (we shall suppose that z is always zero because there is no force in the z-direction and, if there is no initial velocity v_z, there will be nothing to make z other than zero), the force is directed along the line joining the planet to the sun, as shown in Fig. 9–5.

From this figure we see that the horizontal component of the force is related to the complete force in the same manner as the horizontal distance x is to the complete hypotenuse r, because the two triangles are similar. Also, if x is positive,

F_x is negative. That is, $F_x/|F| = -x/r$, or $F_x = -|F|x/r = -GMmx/r^3$. Now we use the dynamical law to find that this force component is equal to the mass of the planet times the rate of change of its velocity in the x-direction. Thus we find the following laws:

$$\begin{aligned} m(dv_x/dt) &= -GMmx/r^3, \\ m(dv_y/dt) &= -GMmy/r^3, \\ r &= \sqrt{x^2 + y^2}. \end{aligned} \tag{9.17}$$

This, then, is the set of equations we must solve. Again, in order to simplify the numerical work, we shall suppose that the unit of time, or the mass of the sun, has been so adjusted (or luck is with us) that $GM \equiv 1$. For our specific example we shall suppose that the initial position of the planet is at $x = 0.500$ and $y = 0.000$, and that the velocity is all in the y-direction at the start, and is of magnitude 1.6300. Now how do we make the calculation? We again make a table with columns for the time, the x-position, the x-velocity v_x, and the x-acceleration a_x; then, separated by a double line, three columns for position, velocity, and acceleration in the y-direction. In order to get the accelerations we are going to need Eq. (9.17); it tells us that the acceleration in the x-direction is $-x/r^3$, and the acceleration in the y-direction is $-y/r^3$, and that r is the square root of $x^2 + y^2$. Thus, given x and y, we must do a little calculating on the side, taking the square root of the sum of the squares to find r and then, to get ready to calculate the two accelerations, it is useful also to evaluate $1/r^3$. This work can be done rather easily by using a table of squares, cubes, and reciprocals: then we need only multiply x by $1/r^3$, which we do on a slide rule.

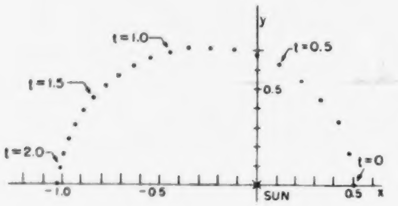


Fig. 9–6. The calculated motion of a planet around the sun.

Our calculation thus proceeds by the following steps, using time intervals $\epsilon = 0.100$: Initial values at $t = 0$:

$$x(0) = 0.500 \qquad y(0) = 0.000$$
$$v_x(0) = 0.000 \qquad v_y(0) = +1.630$$

From these we find:

$$r(0) = 0.500 \qquad 1/r^3(0) = 8.000$$
$$a_x = -4.000 \qquad a_y = 0.000$$

Thus we may calculate the velocities $v_x(0.05)$ and $v_y(0.05)$:

$$\begin{aligned} v_x(0.05) &= 0.000 - 4.000 \times 0.050 = -0.200; \\ v_y(0.05) &= 1.630 + 0.000 \times 0.100 = 1.630. \end{aligned}$$

Now our main calculations begin:

$$\begin{aligned} x(0.1) &= 0.500 - 0.20 \times 0.1 &&= 0.480 \\ y(0.1) &= 0.0 + 1.63 \times 0.1 &&= 0.163 \\ r &= \sqrt{0.480^2 + 0.163^2} &&= 0.507 \\ 1/r^3 &= 7.67 && \\ a_x(0.1) &= 0.480 \times 7.67 &&= -3.68 \\ a_y(0.1) &= -0.163 \times 7.70 &&= -1.256 \\ v_x(0.15) &= -0.200 - 3.68 \times 0.1 &&= -0.568 \\ v_y(0.15) &= 1.630 - 1.26 \times 0.1 &&= 1.505 \\ x(0.2) &= 0.480 - 0.568 \times 0.1 &&= 0.423 \\ y(0.2) &= 0.163 + 1.50 \times 0.1 &&= 0.313 \\ &\text{etc.} && \end{aligned}$$

In this way we obtain the values given in Table 9–2, and in 20 steps or so we have chased the planet halfway around the sun! In Fig. 9–6 are plotted the x- and y-coordinates given in Table 9–2. The dots represent the positions at the succession of times a tenth of a unit apart; we see that at the start the planet moves rapidly and at the end it moves slowly, and so the shape of the curve is determined. Thus we see that we *really do* know how to calculate the motion of planets!

Now let us see how we can calculate the motion of Neptune, Jupiter, Uranus, or any other planet. If we have a great many planets, and let the sun move too, can we do the same thing? Of course we can. We calculate the force on a particular planet, let us say planet number i, which has a position x_i, y_i, z_i ($i = 1$ may represent the sun, $i = 2$ Mercury, $i = 3$ Venus, and so on). We must know the positions of all the planets. The force acting on one is due to all the other bodies which are located, let us say, at positions x_j, y_j, z_j. Therefore the equations are

$$\begin{aligned} m_i \frac{dv_{ix}}{dt} &= \sum_{j=1}^{N} -\frac{Gm_im_j(x_i - x_j)}{r_{ij}^3}, \\ m_i \frac{dv_{iy}}{dt} &= \sum_{j=1}^{N} -\frac{Gm_im_j(y_i - y_j)}{r_{ij}^3}, \\ m_i \frac{dv_{iz}}{dt} &= \sum_{j=1}^{N} -\frac{Gm_im_j(z_i - z_j)}{r_{ij}^3}. \end{aligned} \tag{9.18}$$

Table 9–2

Solution of $dv_x/dt = -x/r^3$, $dv_y/dt = -y/r^3$, $r = \sqrt{x^2 + y^2}$.

Interval: $\epsilon = 0.100$

Orbit $v_y = 1.63$ $v_x = 0$ $x = 0.5$ $y = 0$ at $t = 0$

t	x	v_x	a_x	y	v_y	a_y	r	$1/r^3$
0.0	0.500		−4.00	0.000		0.00	0.500	8.000
		−0.200			1.630			
0.1	0.480		−3.68	0.163		−1.25	0.507	7.675
		−0.568			1.505			
0.2	0.423		−2.91	0.313		−2.15	0.526	6.873
		−0.859			1.290			
0.3	0.337		−1.96	0.442		−2.57	0.556	5.824
		−1.055			1.033			
0.4	0.232		−1.11	0.545		−2.62	0.592	4.81
		−1.166			0.771			
0.5	0.115		−0.453	0.622		−2.45	0.633	3.942
		−1.211			0.526			
0.6	−0.006		+0.020	0.675		−2.20	0.675	3.252
		−1.209			0.306			
0.7	−0.127		+0.344	0.706		−1.91	0.717	2.712
		−1.175			0.115			
0.8	−0.245		+0.562	0.718		−1.64	0.758	2.296
		−1.119			−0.049			
0.9	−0.357		+0.705	0.713		−1.41	0.797	1.975
		−1.048			−0.190			
1.0	−0.462		+0.796	0.694		−1.20	0.834	1.723
		−0.968			−0.310			
1.1	−0.559		+0.858	0.663		−1.02	0.867	1.535
		−0.882			−0.412			
1.2	−0.647		+0.90	0.622		−0.86	0.897	1.385
		−0.792			−0.499			
1.3	−0.726		+0.92	0.572		−0.72	0.924	1.267
		−0.700			−0.570			
1.4	−0.796		+0.93	0.515		−0.60	0.948	1.173
		−0.607			−0.630			
1.5	−0.857		+0.94	0.452		−0.50	0.969	1.099
		−0.513			−0.680			
1.6	−0.908		+0.95	0.384		−0.40	0.986	1.043
		−0.418			−0.720			
1.7	−0.950		+0.95	0.312		−0.31	1.000	1.000
		−0.323			−0.751			
1.8	−0.982		+0.95	0.237		−0.23	1.010	0.970
		−0.228			−0.773			
1.9	−1.005		+0.95	0.160		−0.15	1.018	0.948
		−0.113			−0.778			
2.0	−1.018		+0.96	0.081		−0.08	1.021	0.939
		−0.037			−0.796			
2.1	−1.022		+0.95	0.001		0.00	1.022	0.936
		+0.058			−0.796			
2.2	−1.016		+0.96	−0.079		+0.07	1.019	0.945
					−0.789			
2.3								

Crossed x-axis at 2.101 sec, ∴ period = 4.20 sec.

$v_x = 0$ at 2.086 sec.

Cross x at 1.022, ∴ semimajor axis $= \dfrac{1.022 + 0.500}{2} = 0.761$.

$v_y = 0.796$.

Predicted time $\pi(0.761)^{3/2} = \pi(0.663) = 2.082$.

Further, we define r_{ij} as the distance between the two planets i and j; this is equal to

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \qquad (9.19)$$

Also, $\sum$ means a sum over all values of j—all other bodies—except, of course, for $j = i$. Thus all we have to do is to make more columns, *lots* more columns. We need nine columns for the motions of Jupiter, nine for the motions of Saturn, and so on. Then when we have all initial positions and velocities we can calculate all the accelerations from Eq. (9.18) by first calculating all the distances, using Eq. (9.19). How long will it take to do it? If you do it at home, it will take a very long time! But in modern times we have machines which do arithmetic very rapidly; a very good computing machine may take 1 microsecond, that is, a millionth of a second, to do an addition. To do a multiplication takes longer, say 10 microseconds. It may be that in one cycle of calculation, depending on the problem, we may have 30 multiplications, or something like that, so one cycle will take 300 microseconds. That means that we can do 3000 cycles of computation per second. In order to get an accuracy, of, say, one part in a billion, we would need 4×10^5 cycles to correspond to one revolution of a planet around the sun. That corresponds to a computation time of 130 seconds or about two minutes. Thus it take only two minutes to follow Jupiter around the sun, with all the perturbations of all the planets correct to one part in a billion, by this method! (It turns out that the error varies about as the square of the interval ϵ. If we make the interval a thousand times smaller, it is a million times more accurate. So, let us make the interval 10,000 times smaller.)

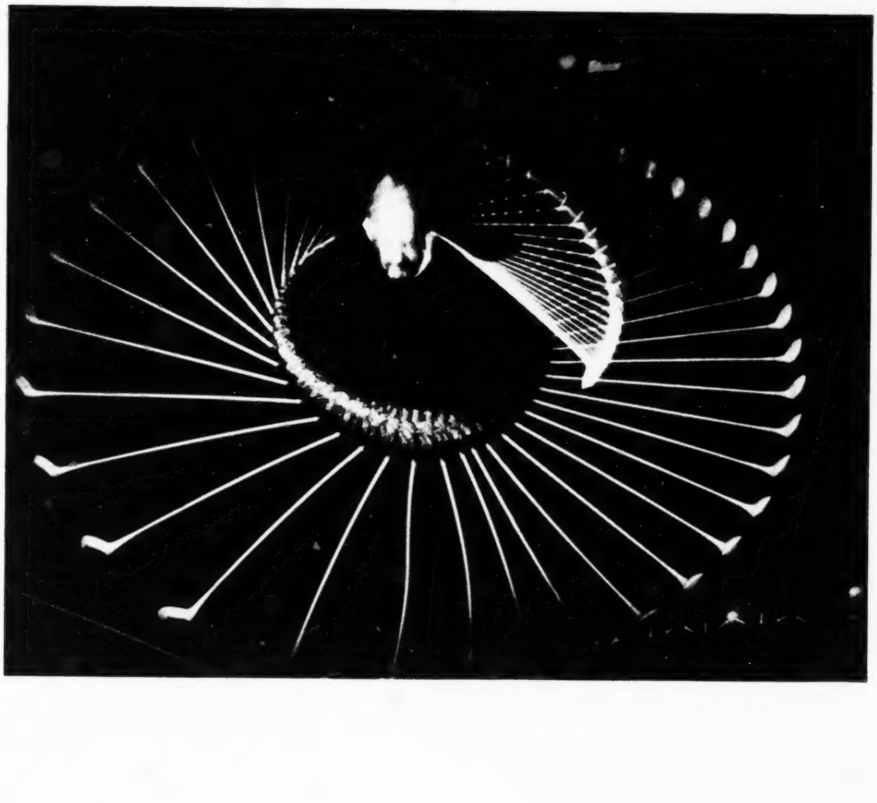
So, as we said, we began this chapter not knowing how to calculate even the motion of a mass on a spring. Now, armed with the tremendous power of Newton's laws, we can not only calculate such simple motions but also, given only a machine to handle the arithmetic, even the tremendously complex motions of the planets, to as high a degree of precision as we wish!

An experimental study of a complex motion, that of a golf club, is outlined. If you do not have a slow-motion movie camera, similar measurements can be made using the stroboscopic picture.

13 The Dynamics of a Golf Club

C. L. Stong

1964



With the aid of a slow-motion movie camera and a co-operative friend any golf player can easily explore the dynamics of his club head during the split second of the drive that separates the sheep from the goats of golfdom. The procedure, as applied by Louis A. Graham, a consulting engineer in Naples, Fla., analyzes the travel of the club head throughout the swing, including its velocity and acceleration at the critical moment of impact—factors that determine whether a squarely struck ball will merely topple off the tee or go a history-making 445 yards to match the performance of E. C. Bliss in August, 1913.

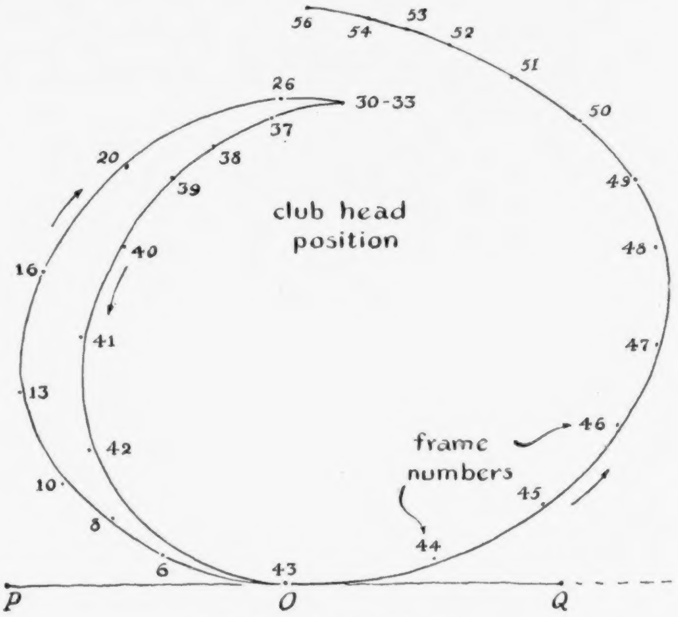
"The procedure is essentially simple," writes Graham, "but the reliability of the results will reflect the care with which certain measurements are made. I pick a sunny day for the experiment and, having arrived at the golf course with my co-operative friend and accessories, tee my ball. Then I place a tee marker precisely four feet in front of the ball and another four feet behind it to make a line that points toward the first green. My friend stations the tripod-mounted camera for a medium close-up shot on a line that intersects the ball at right angles to the tee markers. I address the ball, facing the camera. My friend photographs the complete drive from address to follow-through at the rate of 48 frames per second. The known distance between the tee markers and their position in relation to the club head scales the pictures with respect to distance. The exposure rate—the number of frames per second—of the camera provides the time dimension. (If the exposure rate is not known accurately, it can be calibrated by photographing a phonograph turntable marked with a chalk line and turning at 45 or 78 revolutions per minute.)

"The film is developed and analyzed. One can use either a film-editing device that projects an enlarged image of each frame or a set of enlarged prints of each frame, mounted serially and numbered for identification.

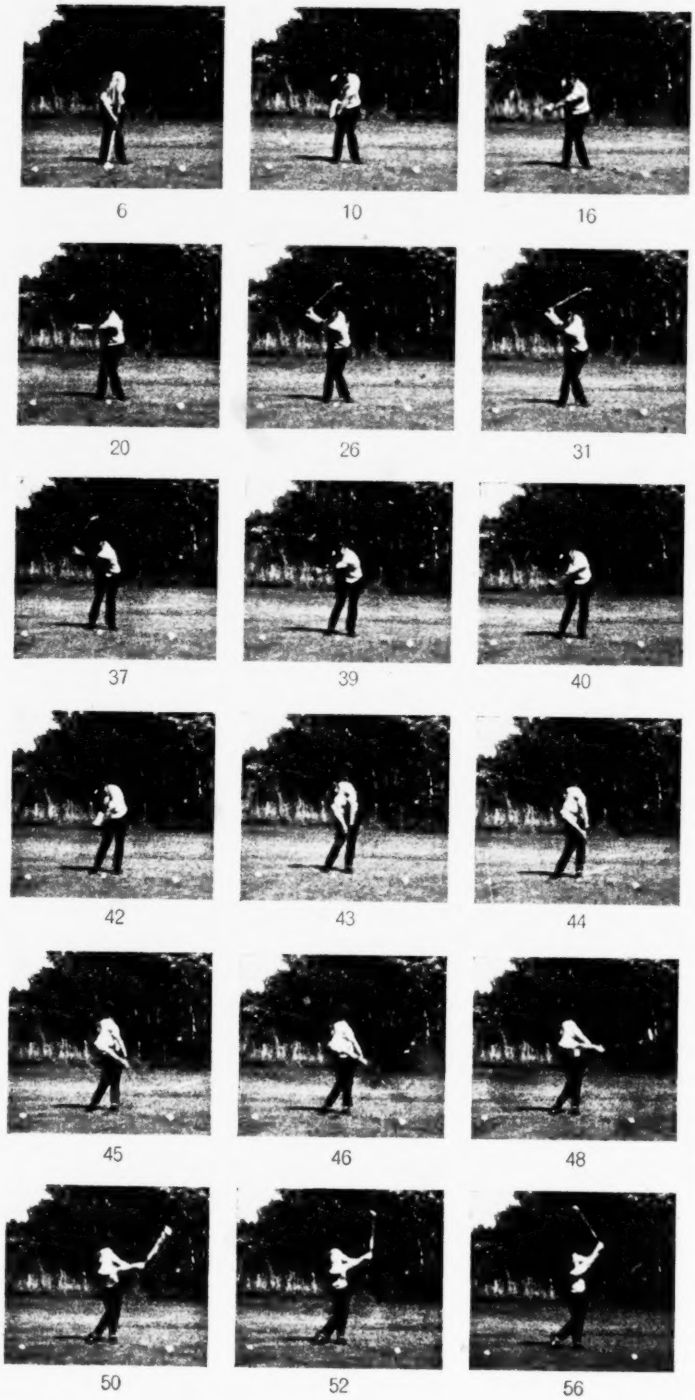
"The next step is to plot the position of the club head during the course of the swing. Since a point in a plane is determined by its distance from two other known points, the position of the club head can be plotted in relation to that of the two tee markers [*see illustration below*]. First, I draw a base line near the bottom of a sheet of graph paper ruled with rectangular co-ordinates and on it locate three equally spaced points: the tee marker *P*, the ball (*O*) and the tee marker *Q*. I usually space these points four inches apart, thus establishing a scale of 12 inches of club head travel per inch of graph paper.

"The location of the club head (*C*) with respect to that of the tee markers can be transferred to the graph by one of three methods. Proportional dividers are handy for transferring the scaled distance from *P* to *C* and from *C* to *Q*. Alternatively, the angles *CPQ* and *CQP* can be measured with a protractor and reconstructed on the graph, point *C* being located at the intersection of lines projected from *P* and *Q*. If no protractor is at hand, the vertical and horizontal distances between *C*, *P* and *Q* can be measured with a square and ruler and similarly transferred to the graph.

"Plot enough points to establish a reasonably smooth track, skipping several frames during slow portions of the



Graph of successive club head positions



6 10 16

20 26 31

37 39 40

42 43 44

45 46 48

50 52 56

Selected frames from slow-motion film of a golf swing

swing. The resulting graph is of course not extremely accurate. The plane in which the club head swings, for example, is inclined to the plane of the film. The track plotted from the image therefore differs slightly from the true excursion of the club head, but the error is not large and can be ignored. By the same token, the travel of the club head from point to point is subsequently measured along straight lines, whereas the club head actually follows a curved path. Error introduced by this source can be minimized by speeding up the camera. My camera, an inexpensive one, is limited to a maximum speed of 48 frames per second, a rate that records the event adequately for the objectives of this experiment.

"The total distance traveled by the club head and its velocity and acceleration are derived from a second set of graphs prepared from the graph of club head position. On a second sheet of graph paper ruled with rectangular coordinates divide the abscissa into a series of uniform increments equal to the total number of frames occupied by the swing and note the corresponding time intervals in seconds as well as the frame numbers. The ordinate will carry two scales: club head travel in feet and club head speed in miles per hour. The scales of the ordinate should provide for a total club head travel of 36 feet and a maximum velocity of about 80 miles per hour. Graphs of convenient proportion result when the length of the ordinate representing 36 feet equals the length representing one second on the abscissa. The maximum velocity of 80 miles per hour need not occupy more than half of the ordinate scale, as shown in the accompanying graph [*upper illustration on opposite page*].

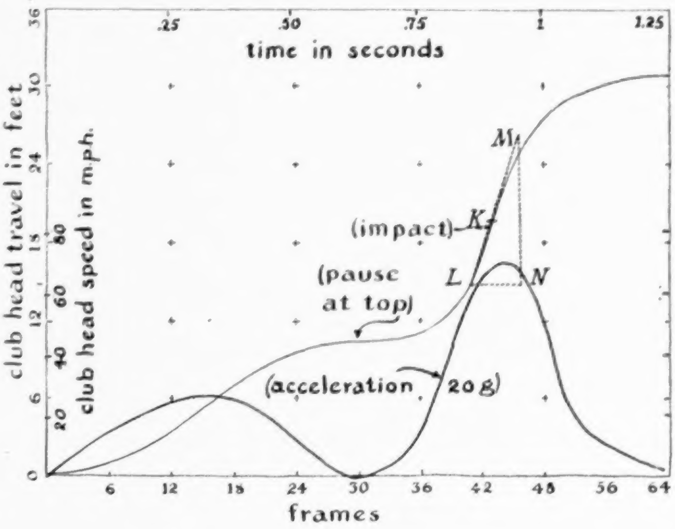
"Data for plotting club head travel against time are derived by measuring the graph of club head position. Make a table of three columns, for frame number, time and distance. Beginning with the point on the graph of club head travel that shows the head addressing the ball, scale the distance to the next point and convert to equivalent feet by referring the measurement to the base line that includes *P*, *O* and *Q*. Measure and tabulate the remaining position points in the same way. When the table is complete, add the distance increments progressively, plot distance against time and draw a smooth curve through the points.

"The speed of the club head at any point is found from this graph by the familiar graphical method of slopes. To find the speed of the club head at about the point of impact (frame No. 43), draw a tangent *LKM* of arbitrary length through *K*. The sides *MN* and *LN* are found by referring to the scale to equal 11.2 feet and .11 second respectively. The speed of the club head at this instant is equal to the ratio 11.2/.11, or 102 feet per second. The result can be expressed in miles per hour by multiplying it by the number of seconds per hour and dividing the product by the number of feet per mile: 102 × 3,600/5,280 = 70 miles per hour. Repeat the procedure for each of the frames, tabulate the results, plot speed versus time and draw a smooth curve through the points.

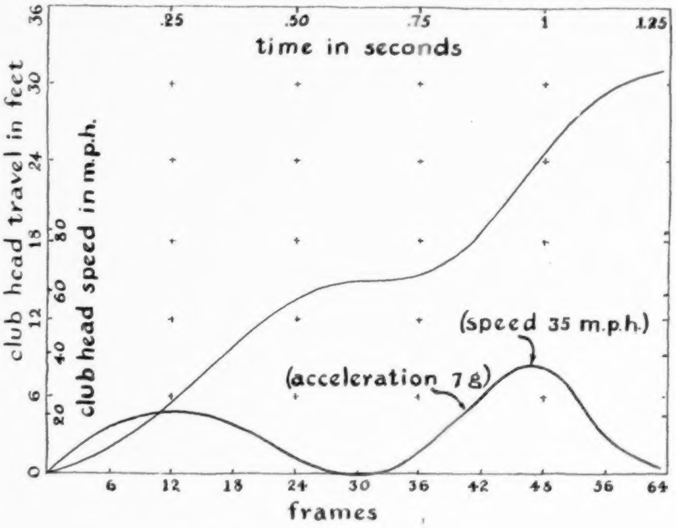
"Club head acceleration can be graphed in the same way or merely computed from the graph of club head speed at frames of particular interest, such as the frame showing the moment of impact. For example, to determine the acceleration of the club head depicted by frame No. 38, draw a tangent to the graph at this point. Then, at some arbitrary point above, say at the point corresponding to a velocity of 56 miles per hour, drop a perpendicular *MN* from the tangent. At another arbitrary point below, say at the point corresponding to a velocity of 12 miles per hour, draw a line *LN* parallel to the abscissa and intersecting both the tangent and *MN*. Inspection of the abscissa discloses that the length *LN* is analogous to a time interval of .1 second. Acceleration is defined as the rate of change of velocity and is equal to the difference between the final velocity and initial velocity divided by the time interval between the two. In this example the velocity difference is 56 miles per hour minus 12 miles per hour, or, expressed in feet per second: (56 − 12) × 5,280/3,600 = 64 feet per second. The acceleration is 64/.1 = 640 feet per second per second. The acceleration of gravity (g) amounts to 32 feet per second per second. The acceleration of the club head at frame No. 38 in terms of g is accordingly 640/32, or 20 g!

"Having performed this rainy-afternoon portion of the procedure, what reward awaits the duffer? For one thing, he can see at a glance why his drives do not match those of a professional golfer. The graphs discussed so far show the performance of golf professional Dick Bull using an iron. His swing from address to follow-through required 1.17 seconds. The club head traveled 31 feet. His backswing occupied .6 second. He paused at the top about .1 second. More interesting than these figures, in my opinion, are those of the club head speed and acceleration Bull achieved: the increase in club head speed during the .1 second before impact from 15 miles per hour to an amazing 70 miles per hour, representing an acceleration of slightly over 20 g. Graphs of Bull's performance with a driver, although different in many respects from those of his irons, show exactly the same figure for speed, 70 miles per hour, and an acceleration of 22 g, a remarkably uniform performance. Similar analysis of the performance of a fairly good amateur using a driver shows precisely half the velocity of Bull's club, 35 miles per hour, and an acceleration at impact of only seven g [*see lower illustration below*].

"Although these methods of analyzing motion are routine in engineering cir-



Speed and acceleration graph for a professional's swing



Similar graph for an amateur's performance

Athletic events involve measurements of distance and time, and so bring in the same error considerations that one also meets in the laboratory.

14 Bad Physics in Athletic Measurements

P. Kirkpatrick

1944

THE physics teacher has been accustomed to find in athletic activities excellent problems involving velocities, accelerations, projectiles and impacts. He has at the same time overlooked a rich source of illustrations of fictitious precision and bad metrology. When the student is told that the height of a tree should not be expressed as 144.632 ft if the length of its shadow has been measured only to the nearest foot, the student may see the point at once and yet ask, "What difference does it make?" But when shown that common procedures in measuring the achievements of a discus thrower could easily award a world's record to the wrong man, the student agrees that good technic in measurement is something more than an academic ideal. The present discussion[1] has been prepared partly to give the physics teacher something to talk about, but also to start a chain of publicity which may ultimately make athletic administrators better physicists and so make their awards more just.

If physicists were given charge of the measurements of sport, one may feel sure that they would frown upon the practice of announcing the speed of a racing automobile in six or seven digits—see, for example, the *World Almanac* for any year—when neither the length of the course nor the elapsed time is known one-tenth so precisely. They could and would point out such inconsistencies as that observed in some of the events of the 1932 Olympic games when races were electrically and photographically timed to 0.01 sec, but with the starting gun fired from such a position that its report could not reach the ears of the waiting runners until perhaps 0.03 to 0.04 sec after the official start of the race. In this case, electric timing was used only as an unofficial or semi-official supplement to 0.1-sec hand timing; but it is easy to see that a systematic error of a few hundredths of a second will frequently cause stopwatch timers to catch the wrong tenth.

Scientific counsel on the field would immediately advise judges of the high jump and pole vault to measure heights from the point of take-off instead of from an irrelevant point directly below the bar which should be at the same level but sometimes isn't. Physicists would suggest equipping field judges with surveying instruments for determining after each throw, not only how far the weight traveled but also the relative elevation of the landing point and the throwing circle. Certainly it is meaningless if not deceptive to record weight throws to a small fraction of an inch when surface irregularities may be falsifying by inches the true merit of the performance.

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[1] Some of the material in this article appeared in a paper by the author in *Scientific American*, April 1937, and is incorporated here by permission of the editors.

In shot-putting, for example, a measured length will be in error by practically the same amount as the discrepancy between initial and final elevations, since the flight of the shot at its terminus is inclined at about 45° to the horizontal. For the discus the effect is some three times as serious because of the flatter trajectory employed with this missile, while broad jumpers under usual conditions must be prepared to give or take as much as 0.5 ft, according to the luck of the pit. Meanwhile, the achievements in these events go down in the books with the last eighth or even the last sixteenth of an inch recorded.

At the 1932 Olympic Games an effective device was used to grade the broad-jumping pit to the level of the take-off board before each leap, but the practice has not become general. Athletic regulations, indeed, recognize the desirability of proper leveling in nearly all the field events, but in actual usage not enough is done about it. Since sprinters are not credited with records achieved when blown along before the wind, there is no obvious reason why weight hurlers should be permitted to throw things down hill.

The rule books make no specification as to the hardness of the surface upon which weights shall be thrown, but this property has a significant effect upon the measured ranges of the shot and hammer, since it is prescribed that measurement shall be made to the near side of the impression produced by the landing weight. In a soft surface this impression may be enlarged in the backward direction enough to diminish the throw by several times the ostensible precision of the measurement.

A physicist would never check the identity of three or four iron balls as to mass by the aid of grocers' scales or the equivalent and then pretend that there was any significance in the fact that one of them was thrown a quarter of an inch farther than the others. In measuring the length of a javelin throw, no physicist who wanted to be right to ¼ in. would be content to establish his perpendicular from the point of fall to the scratchline by a process of guesswork, but this is the way it is always done by field judges, even in the best competition.

Among the numerous errors afflicting measurements in the field sports, there is none which is more systematically committed, or which could be more easily rectified, than that pertaining to the variation of the force of gravity. The range of a projectile dispatched at any particular angle of elevation and with a given initial speed is a simple function of g. Only in case the end of the trajectory is at the same level as its beginning does this function become an inverse proportionality; but in any case the relationship is readily expressed, and no physicist will doubt that a given heave of the shot will yield a longer put in equatorial latitudes than it would in zones where the gravitational force is stronger. Before saying that the 55-ft put achieved by A in Mexico City is a better performance than one of 54 ft, 11 in. which B accomplished in Boston, we should surely inquire about the values of g which the respective athletes were up against, but it is never done. As a matter of record, the value of g in Boston exceeds that in Mexico City by ¼ percent, so the shorter put was really the better. To ignore the handicap of a larger value of g is like measuring the throw with a stretched tape. The latter practice would never be countenanced under AAU or Olympic regulations, but the former is standard procedure.

Rendering justice to an athlete who has had to compete against a high value of g involves questions that are not simple. It will be agreed that he is entitled to some compensation and that in comparing two throws made under conditions similar except as to g, the proper procedure would be to compare not the actual ranges achieved, but the ranges which would have been achieved had some "standard" value of g—say 980 cm/sec^2—prevailed in both cases. The calculation of exactly what would have happened is probably impossible to physics. Although it is a simple matter to discuss the behavior of the implement after it leaves the thrower's hand and to state how this behavior depends upon g, the dependence of the initial velocity of projection upon g depends upon the thrower's form and upon characteristics of body mechanics to which but little attention has so far been devoted.

The work done by the thrower bestows upon the projectile both potential and kinetic energy. In a strong gravitational field, the imparted potential energy is large and one must therefore suppose the kinetic energy to be reduced, since the thrower's propelling energy must be distributed to both. We have no proof, however, that the ***total*** useful work is constant despite variation of g, nor do we know the manner of its inconstancy, if any. The muscular catapult is not a spring, subject to Hooke's law, but a far more complicated system with many unknown characteristics. The maximum external work which one may do in a single energetic shove by arms, legs or both obviously depends partly upon the resisting force encountered. Only a little outside work can be done in putting a ping-pong ball because the maximum possible acceleration, limited by the masses and other characteristics of the bodily mechanism itself, is too slight to call out substantial inertial forces in so small a mass. The resisting force encountered when a massive body is pushed in a direction that has an upward component, as in shot-putting, does of course depend upon g; and until we know from experiment how external work in such an effort varies with resisting force, we shall not be able to treat the interior ballistics of the shot-putter with anything approaching rigor.

Several alternative assumptions may be considered. If we suppose that the ***velocity*** of delivery, or "muzzle velocity," v, of the missile is unaffected by variations of g, we have only the external effect to deal with. Adopting the approximate range formula $R=v^2/g$ (which neglects the fact that the two ends of the trajectory are at different levels and which assumes the optimum angle of elevation) we find that the increment of range $\mathrm{d}R$ resulting from an increment $\mathrm{d}g$ is simply $-R\mathrm{d}g/g$. On the more plausible assumption that the ***total work done on the projectile*** is independent of g, this total to include both the potential and kinetic energies imparted, one obtains as a correction formula,

$$\mathrm{d}R=-\left(1+\frac{2h}{R}\right)R\frac{\mathrm{d}g}{g}, \tag{1}$$

where h is the vertical lift which the projectile gets while in the hand of the thrower. A third assumption, perhaps the most credible of all, would hold constant and independent of g the ***total work done upon the projectile and upon a portion of the mass of the thrower's person.*** It is not necessary to decide how much of the thrower's mass goes into this latter term; it drops out and we have again Eq. (1), provided only that the work done on the thrower's body can be taken into account by an addition to the mass of the projectile.

These considerations show that a variation of g affects the range in the same sense before and after delivery, an increase in g reducing the delivery velocity and also pulling the projectile down more forcibly after its flight begins. They indicate also that the latter effect is the more important since, in Eq. (1), $1>2h/R$ by a factor of perhaps five in the shot-put and more in the other weight-throwing events.

One concludes that the ***least*** which should be done to make amends to a competitor striving against a large value of g is to give him credit for the range which his projectile would have attained, for the same initial velocity, at a location where g is "standard." This is not quite justice, but it is a major step in the right direction. The competitor who has been favored by a small value of g should of course have his achievement treated in the same way.

The corrections so calculated will not be negligible magnitudes, as Fig. 1 shows. They are extremely small percentages of the real ranges, but definitely exceed the ostensible probable errors of measurement. It is not customary to state probable errors explicitly in connection with athletic measurements, but when a throw is recorded as 57 ft, $1\frac{3}{32}$ in., one naturally concludes that the last thirty-second inch, if not completely reliable, must have been regarded as having ***some*** significance.

ROTATION OF THE EARTH

It is customary to take account of the effects of terrestrial rotation when aiming long-range guns, but athletes and administrators of sport have given little or no attention to such effects in relation to their projectiles. As a matter of fact they should, for at low latitudes the range of a discus or shot thrown in an eastward direction

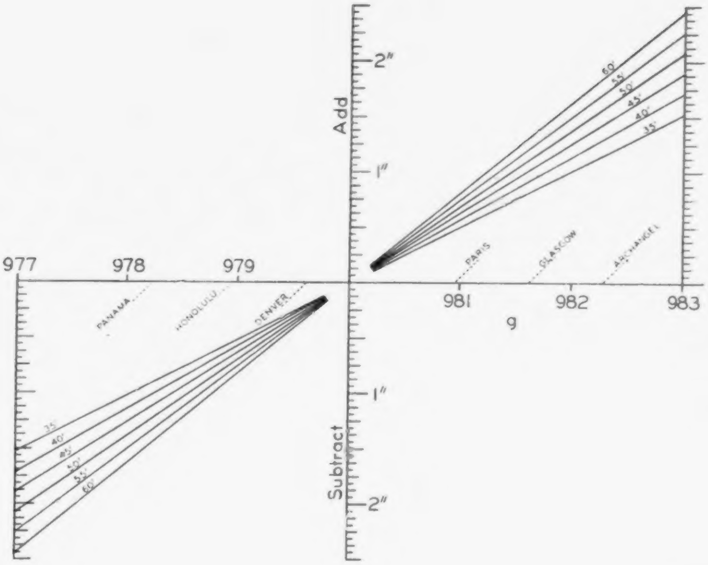

FIG. 1. Graphs for normalizing shot-put ranges to the common value $g=980$ cm/sec^2. Ranges achieved where $g=980$ cm/sec^2 are not in need of adjustment, but a range of 50 ft (see inclined line marked 50′) achieved at Glasgow, where $g=981.6$ cm/sec^2, is entitled to a premium of 1⅛ in. which should be added before comparing the put with one achieved elsewhere. Distances accomplished where $g<980$ cm/sec^2 should be subjected to the deductions indicated by graphs in the third quadrant.

exceeds that of a westward throw by more than the ostensible precision of such measurements. The difference between the range of a projectile thrown from the surface of the real earth and the range of one thrown from a nonrotating earth possessing the same local value of g is given by[2]

$$\text{Range}=\frac{V_0^2\sin 2\alpha}{g}+\frac{4\omega V_0^3}{3g^2}\times\sin\alpha[4\cos^2\alpha-1]\cos\lambda\sin\mu, \quad (2)$$

where g is the ordinary acceleration due to weight, V_0 is the initial speed of the projectile, α is the angle of elevation of initial motion (measured upward from the horizontal in the direction of projection), ω(rad/sec) is the angular speed of rotation of the earth, λ is the geographic latitude of the point of departure of the projectile, and μ is the azimuth of the plane of the trajectory, measured clockwise from the north point.

A derivation of this equation (though not the first) is given in reference 2, along with a discussion of its application to real cases. The approximations accepted in the derivation are such as might possibly be criticized where long-range guns are considered, but they introduce no measurable errors into the treatment of athletic projectiles.

The first term of the right-hand member of Eq. (2) is the ordinary elementary range expression, and naturally it expresses almost the whole of the actual range. The second term is a small correction which is of positive sign for eastbound projectiles ($0<\mu<180°$) and negative for westbound. The correction term, being proportional to V_0^3, increases with V_0 at a greater rate than does the range as a whole. Hence the *percentage* increase or decrease of range, because of earth rotation, varies in proportion to V_0 or to the square root of the range itself. Evidently this effect is a maximum at the equator and zero at the poles. Inspection of the role of α shows that the correction term is a maximum for a 30° angle of elevation and that it vanishes when the angle of elevation is 60°.

By the appropriate numerical substitutions in Eq. (2), one may show that a well-thrown discus in tropic latitudes will go an inch farther eastward than westward. This is many times the apparent precision of measurement for this event, and records have changed hands on slimmer margins. Significant effects of the same kind, though of lesser magnitude, appear in the cases of the javelin, hammer, shot and even the broad jump, where the east-west differential exceeds the commonly recorded sixteenth of an inch.

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[2] P. Kirkpatrick, Am. J. Phys. 11, 303 (1943).

Figures 1 and 2 are types of correction charts that might be used to normalize the performances of weight throwers to a uniform value of g and a common direction of projection. Figure 1 has been prepared with the shot-put in mind, but is not restricted to implements of any particular mass. The inclined straight lines of this figure are graphs of $-\mathrm{d}R$ *versus* $\mathrm{d}g$ from Eq. (1). Values of the parameter R are indicated on the graphs. The uniform value 100 cm has been adopted for h, an arbitrary procedure but a harmless one in view of the insensitivity of $\mathrm{d}R$ to h.

Figure 2, particularly applicable to the hammer throw, furnishes means for equalizing the effect of earth spin upon athletes competing with the same implement but directing their throws variously as may be necessitated by the lay-out of their respective fields. An angle of elevation of 45° has been assumed in the construction of these curves, a somewhat restrictive procedure which finds justification in the fact that no hammer thrown at an angle significantly different from 45° is likely to achieve a range worth correcting. These curves are plotted from Eq. (2); their application to particular cases is described in the figure legend.

Upon noticing that some of these corrections are quite small fractions of an inch, the reader may ask whether the trouble is worth while. This is a question that is in great need of clarification and one that may not be answered with positiveness until the concept of the probable error of a measurement shall have become established among the metrologists of sport. Physicists will agree that to every measurement worth conserving for the attention of Record Committees should be attached a statement of its probable error; without such a statement there will always be the danger of proclaiming a new record on the basis of a new performance that is apparently, though not really, better than the old. If the corrections of Fig. 2 exceed the probable error to be claimed for a measurement, then those corrections must be applied.

The aim of the American Athletic Union in these matters is hard to determine. Watches must be "examined," "regulated" and "tested" by a reputable jeweler or watchmaker, but one finds no definition of what constitutes an acceptable job of regulation. Distances must be measured with "a steel tape." The Inspector of

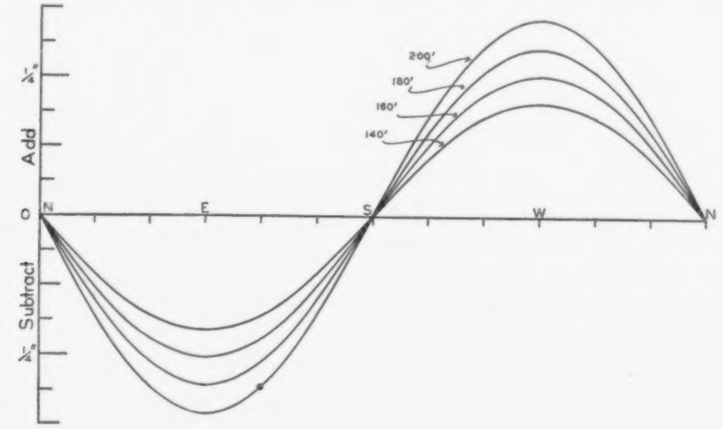

FIG. 2. Curves for rendering throws in various directions comparable. The assumed latitude is 30°, either north or south, and the assumed angle of elevation is 45°. Since the range has a maximum for about this angle of elevation, the curves also apply well to angles several degrees on either side. The curves show, for example (circled point), that a missile thrown 200 ft in a direction 30° south of east should have $\frac{3}{16}$ in. subtracted from its range in order to bring it into fair comparison with unadjusted northward or southward throws or with throws in any other direction which have been adjusted by reference to curves of this type appropriately constructed for their respective latitudes.

Implements must find the weights of the implements "correct." Such ideals of perfection are not realistic, and the only alternative is to recognize the existence of error and state its magnitude. The minimum permissible weight for each implement is prescribed both in pounds and in kilograms by AAU rules, but in no instance are the prescriptions exactly equivalent. A discus thrower whose implement just satisfies the metric specification will use a discus 4 gm, or $\frac{1}{5}$ percent, lighter than that of a competitor whose discus just passes as judged by an inspector using perfect scales calibrated in British units. Those 4 gm will give the former athlete two or three extra inches of distance, an advantage that might be decisive.

Similar comments could be made about the rules of competition of the ICAAAA, where one reads that the javelin throw is measured from the point at which the point of the javelin first strikes the ground. This is a mark that cannot in general be determined to the often implied $\frac{1}{8}$ in. since it is obliterated by the subsequent penetration of the implement. Any javelin throw as correctly measured by ICAAAA rules will show a greater distance than if measured by AAU rules, but few field judges know this nor could they do much about it if they did. It is probable that the rules do not say what was meant in these cases. It is interesting that whereas the hammer, shot and discus must be thrown upon a level surface, there is no such requirement in the case of the javelin.

Any serious attempt to put the measurements of sport upon a scientific basis would be met with vast inertia if not positive hostility. The training of athletes is still very largely an art, and there is no reason to suppose that those who are at present practicing this art with success will be predisposed to changes involving ways of thought which, however commonplace in other disciplines, are novel in athletic competition. One eminent track and field coach, a producer of national, Olympic and world champions, told the writer that he had no interest in hairsplitting; that leveling the ground accurately would be too much trouble; that common sense is better than a wind gage for estimating the effect of wind conditions on sprinters; that a man can't put the shot by theory—it's all in the feeling; that the exact angle of elevation is unimportant as long as he gets it in the groove.

A few years ago, the writer published some criticisms along the lines of the present article and sent reprints to each of the several hundred National Committeemen of the AAU. One acknowledgment was received, but no reactions to the subject matter. In a sense, this indifference was only just recompense for the writer's habit of ignoring communications from nonphysicists proposing novel theories of the atom, or otherwise instructing the physicist as to the foundations of his science.

There probably exists a general feeling that part of the charm of sport resides in accident and uncertainty. Any discussion of the possibility of replacing the balls-and-strikes umpire in baseball by a robot will bring out the opinion that the fallibilities of the umpire are part of the entertainment for which the public pays. An optical instrument for determining from the sidelines whether or not a football has been advanced to first down was tried out in California a few years ago. It was technically successful, but a popular failure. The crowd was suspicious of a measurement that it did not understand and could not watch; the players begrudged the elimination of the breather which a chain measurement affords; and even the linemen protested the loss of their dramatic moment.

Though entertained by such attitudes, the physicist will hardly be able to dismiss a feeling that in any field of popular importance or interest, it is improper to keep up the appearances of accurate and comparable measurement without doing what might be done to gain the reality. In the matter of athletic records, he and very few others know what to do about it.[3]

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[3] The author will be pleased to furnish reprints of this article to readers who would find interest in bringing it to the attention of athletic authorities.

Observation of nature by Renaissance artists and craftsmen was a precursor of the new scientific outlook. This in turn accelerated technology, leading to the industrial revolution.

15 The Scientific Revolution

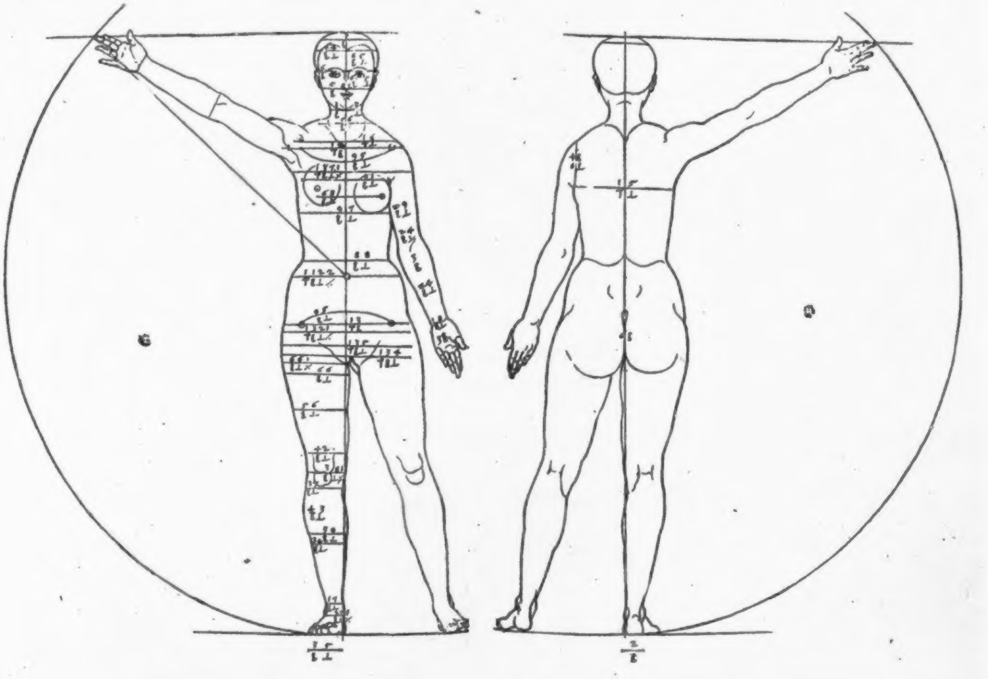
Herbert Butterfield

1960

The preceding article leaves *Homo sapiens* in about 2500 B.C., after his invention of the city-state. Our story does not really get under way until some 4,000 years later. Thus, in turning to the next major revolution in man's impact on his environment, we seem to pass over almost all of recorded human history. No revolution is without its antecedents, however. Although the scientific-industrial age is a recent and original achievement of Western man, it has deep historical roots.

Western civilization is unique in its historical-mindedness as well as in its scientific character. Behind it on the one hand are the ancient Jews, whose religious literature was largely historical, who preached a God of history, and taught that history was moving to a mighty end, not merely revolving in cycles of growth and decay. On the other hand are the ancient Greeks. Their literature has provided a training in logic, a stimulus to the exercise of the critical faculties and a wonderful grounding in mathematics and the physical sciences.

In western Europe civilization had a comparatively late start. For thousands



ANATOMY, studied by Renaissance artists, was the first of the sciences to be placed on a modern footing. This drawing is from a copy of Albrecht Dürer's work *De Symmetria Partium Humanorum Corporum* in the Metropolitan Museum of Art in New York.

of years the lands at the eastern end of the Mediterranean had held the leadership in that whole section of the globe. It was in the West, furthermore, that the Roman Empire really collapsed, and was overrun by "barbarian invaders." Here much of the ancient culture was lost, and society reverted to comparatively primitive forms. In the meantime a high Byzantine civilization had its center in Constantinople, and a brilliant Arabian one in Baghdad. It would be interesting to know why Western man, though he started late, soon proved himself to be so much more dynamic than the peoples farther to the east.

In the formative period of a civilization religion plays a more important part than we today can easily understand. After the fall of the Roman Empire the comparatively primitive peoples in much of Europe were Christianized by conquest or through royal command; in the beginning it was a case of pagans merely changing the names of their gods. But in the succeeding centuries of the Middle Ages the Church deepened spiritual life and moral earnestness. It became the great educator, recovering ancient scholarship and acting as the patron of the arts. By the 13th century there had developed a lofty culture, very much under the presidency of religion, but a religion that nourished the inner life, stimulated heart-searchings and examinations of conscience and set an eternal value upon each individual soul. The Western tradition acquired a high doctrine of personality.

By the year 1500, when the Renaissance was at its height, the West had begun to take the command of world history. The expansion of Islam had been contained. The terrible Asian hordes, culminating in the Mongols and the Turks, that had overrun the eastern Mediterranean lands had been stopped in central Europe. One of the reasons first for survival and then for progress in the West was its consolidation into something like nation-states, a form of polity more firm and more closely knit than the sprawling Asiatic empires.

Yet the Renaissance belongs perhaps to the old (that is, the medieval) world rather than to the new; it was still greatly preoccupied with the recovery of the lost learning of ancient Greece and Rome. Its primary interest was not in scientific studies, but now, after something like a thousand years of effort, the West had recaptured virtually all it ever was to recover of ancient Greek scholarship and science. Only after this stage had been reached could the really original developments in the study of the physical universe begin. The Western mind was certainly becoming less other-worldly. In the later Middle Ages there was much thought about the nature of man as well as about the nature of God, so that a form of Christian humanism had already been developing. The Renaissance was essentially humanistic, stressing man as the image of God rather than as the doomed sinner, and it installed in western Europe the form of classical education that was to endure for centuries. The philosophy of the time dwelt much on the dignity of man. Our modern Western values therefore have deep historic roots.

GOTHIC CLOCK, dating from the early 16th century, was photographed at the Smithsonian Institution. Stone at bottom is the driving weight; arm at top is part of escapement. Clockworks were among earliest examples of well-ordered machines.

And the men of the Renaissance were still looking backward, knowing that the peak of civilization had been reached in remote antiquity, and then lost. It was easy for them to see the natural process of history as a process of decline.

Signs of something more modern had begun to appear, but they belong chiefly to the realm of action rather than to that of thought. Theories about the universe (about the movements of the planets, for example) had still to be taken over bodily from the great teachers of the ancient world. On the other hand, in action Western man was already proving remarkably free and adventurous: in his voyages of discovery, in the development of mining and metallurgy and in the creative work of the Renaissance artists. Under these conditions scientific thought might make little progress, but technology had been able to advance. And perhaps it was the artist rather than the writer of books who, at the Renaissance, was the precursor of the modern scientist.

The artists had emancipated themselves from clerical influence to a great degree. The Florentine painters, seeking the faithful reproduction of nature, sharpened observation and prepared the way for science. The first of the sciences to be placed on a modern footing—that of anatomy—was one which the artists cultivated and which was governed by direct observation. It was the artists who even set up the cry that one must not be satisfied to learn from the ancients or to take everything from books; one must examine nature for oneself. The artists were often the engineers, the designers of fortifications, the inventors of gadgets, they were nearer to the artisan than were the scholars, and their studios often had the features of a laboratory or workshop. It is not surprising to find among them Leonardo da Vinci—a precursor of modern science, but only a precursor, in spite of his brilliance, because the modern scientific method had not yet emerged.

Records show that in the 15th century a Byzantine scholar drew the attention of his fellow-countrymen to the technological superiority of the West. He mentioned progress in machine saws, shipbuilding, textile and glass manufacture and the production of cast iron. Three other items should be added to the list: the compass, gunpowder and the printing press. Although they might not have originated in Christendom, they had not been handed down from classical antiquity. They came to be the first concrete evidence generally adduced to show that the moderns might even excel the ancients. Before 1500, artillery had assisted the consolidation of government on something like the scale of the nation-state. Printing was to speed up intellectual communication, making possible the wider spread of a more advanced kind of education and facilitating the rise of a lay intelligentsia.

In setting the stage for modern developments the economic situation is of fundamental importance. By this time a high degree of financial organization had been attained. The countryside might look much as it had done for a thousand years, but the Renaissance flourished primarily in the city-states of Italy, the Netherlands and southern Germany, where commerce and industry had made great advances. The forms of economic life were calculated to bring out individual enterprise; and in the cities the influence of priests declined—the lay intelligentsia now took the lead. There had existed greater cities and even an essentially urban civilization in ancient times. What was new was the form of the economic life, which, by the opportunities it gave to countless individuals, possessed dynamic potentialities.

It was a Western world already steeped in humanism that entered upon a great scientific and technological development. But if Western man decided now to take a hand in shaping his own destiny, he did it, as on so many other occasions, only because he had been goaded by problems that had reduced him to desperation. The decisive problems were not material ones, however. They were baffling riddles presented to the intellect.

The authority of ancient scholarship was shaken when it came to be realized that the great Greek physician Galen had been wrong in some of his observations, primarily in those on the heart. In the 16th century successive discoveries about the heart and the blood vessels were made in Padua, culminating a little later in William Harvey's demonstration in England of the circulation of the blood. The whole subject was now set on a right footing, so that a flood of further discoveries was bound to follow very quickly. Harvey's work was of the greatest importance, moreover, because it provided a pattern of what could be achieved by observation and methodical experiment.

The older kind of science came to shipwreck, however, over two problems connected with motion. Aristotle, having in mind a horse drawing a cart, had imagined that an object could not be kept moving unless something was pulling or pushing it all the time. On this view it was difficult to see why projectiles stayed in motion after they had become separated from the original propulsive force. It was conjectured that a flying arrow must be pushed along by the rush of air that its previous motion had created, but this theory had been recognized to be unsatisfactory. In the 16th century, when artillery had become familiar, the student of motion naturally tended to think of the projectile first of all. Great minds had been defeated by this problem for centuries before Galileo altered the whole approach and saw motion as something that continued until something intervened to check it.

A great astronomical problem still remained, and Copernicus did not solve it alone. Accepting the recognized data, he had shown chiefly that the neatest explanation of the old facts was the hypothesis of a rotating earth. Toward the end of the century new appearances in the sky showed that the traditional astronomy was obsolete. They demonstrated that the planets, for example, instead of being fixed to crystalline spheres that kept them in their proper courses, must be floating in empty space. There was now no doubt that comets belonged to the upper regions of the sky and cut a path through what had been regarded as the hard, though transparent, spheres. It was now not easy to see how the planets were held on a regular path. Those who followed Copernicus in the view that the earth itself moved had to face the fact that the science of physics, as it then existed, could not possibly explain how the motion was produced.

In the face of such problems it began to be realized that science as a whole needed renovation. Even in the 16th century people were beginning to examine the question of method. In this case a great historic change was willed in advance and consciously attempted. Men called for a scientific revolution before the change had occurred, and before they knew exactly what the situation demanded. Francis Bacon, who tried to establish the basis for a new scientific method, even predicted the magnitude of its possible consequences —the power that man was going to ac-

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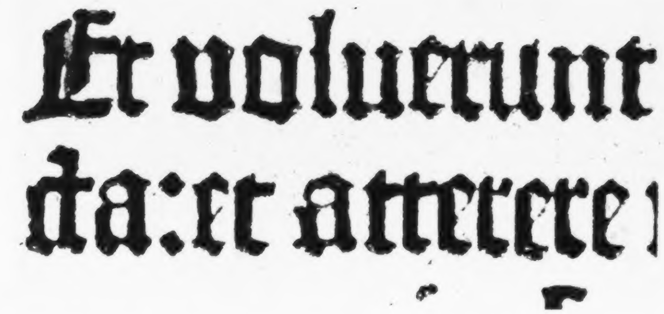
COMPASS ROSE is reproduced from *The Art of Navigation*, published in France in 1666. The invention of the compass, which was not an achievement of classical antiquity, encouraged the men of the Renaissance to believe that they might come to excel the ancients.



MOVABLE TYPE CAST FROM MATRICES was contribution of Johann Gutenberg to art of printing. Sample of his type, enlarged about four diameters, is from his Bible, printed about 1456. Bible in which this type appears is in Pierpont Morgan Library in New York.

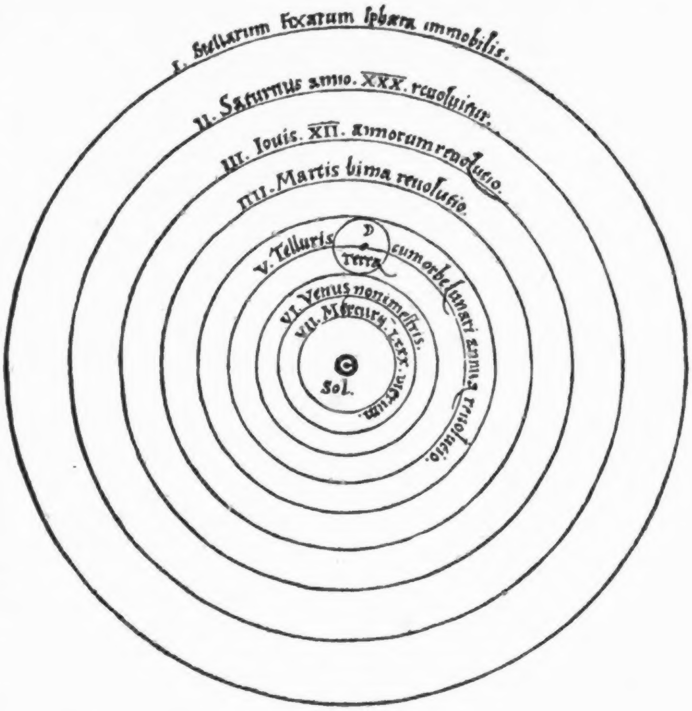
quire over nature. It was realized, furthermore, that the authority of the ancient world, as well as that of the Middle Ages, was in question. The French philosopher René Descartes insisted that thinking should be started over again on a clean slate.

The impulse for a scientific revolution came from the pressure of high intellectual needs, but the tools of civilization helped to give the new movement its direction. In the later Middle Ages men had become more conscious of the existence of the machine, particularly through mechanical clocks. This may have prepared them to change the formulation of their problems. Instead of seeking the "essence" of a thing, they were now more prepared to ask, even of nature, simply: How does it work?

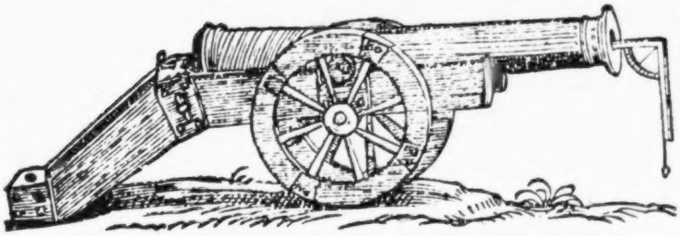
The student of the physical universe, like the artists before him, became more familiar with the workshop, learning manipulation from the artisan. He interested himself in problems of the practical world: artillery, pumps, the determination of longitude. Experimentation had long existed, but it now became more organized and methodical as the investigator became more conscious of what he was trying to do. In the 17th century, moreover, scientific instruments such as the telescope and the microscope came into use.

But theory mattered too. If Galileo corrected a fallacious view of motion, it was because his mind was able to change the formulation of the whole problem. At least as important as his experimentation was his mathematical attack on the problem, which illustrated the potential role of mathematics in the transformation of science.

Another momentous factor in developing the new outlook was the revival of an ancient view: that matter is composed of infinitesimally small particles. This view was now at last presented in a form that seemed consistent with Christianity (because the combinations of the particles which produced the varied world of physical things were no longer regarded as the mere product of chance), so that the atomic theory was able to acquire a wide currency. It led to a better appreciation of the intricate texture of matter, and it proved to be the source of innumerable new hypotheses. The theory seemed to open the way to a purely mechanical explanation of the universe, which should account for everything by the shape, the combination and the motion of the particles. Long before such an explanation had been achieved, men were aspiring to it. Even religious men were arguing that Creation itself would have been imperfect if God had not made a universe that was a perfectly regular machine.


I. Stellarum Fixarum sphæra immobilis.
II. Saturnus anno. XXX. reuoluitur.
III. Iouis. XII. annorum reuolutio.
IIII. Martis bima reuolutio.
V. Telluris cum orbe lunari annua reuolutio.
Terra
VI. Venus nonimestris.
VII. Mercurii. LXXX. dierum.
Sol.

NEW COSMOLOGY OF COPERNICUS placed a fixed sun (*Sol*) at the center of the universe. The sphere of the fixed stars (*I.*) and the spheres of the six known planets revolved around the sun. Circle inscribed around the earth (*Terra*) is the lunar sphere. This woodcut appears in Copernicus's *On the Revolution of the Celestial Spheres* (1543).



TRAJECTORIES OF PROJECTILES were calculated with aid of protractor device (*right*) invented by Niccolò Tartaglia, an Italian engineer and mathematician who died in 1577. Ballistics problems drew attention to the inadequacy of the Aristotelian ideas about motion.

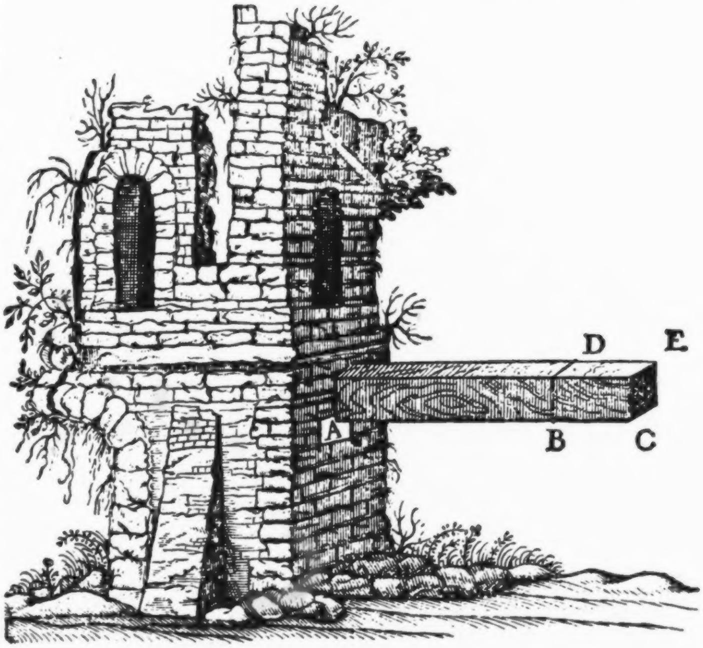
The civilization that had begun its westward shift in the later Middle Ages was moving north and west. At the Renaissance Italy still held the primacy, but with the Reformation the balance shifted more definitely to the north. By the closing decades of the 17th century economic, technological and scientific progress centered on the English Channel. The leadership now belonged to England, France and the Netherlands, the countries that had been galvanized by the commerce arising from the overseas discoveries of the 15th century. And the pace was quickening. Technique was developing apace, economic life was expanding and society was moving forward generally in an exhilarating way.

The solution of the main problems of motion, particularly the motion of the earth and the heavenly bodies, and the establishment of a new notion of scientific method, took a hundred years of effort after the crisis in the later decades of the 16th century. A great number of thinkers settled single points, or made attempts that misfired. In the period after 1660 a host of workers in Paris and London were making science fashionable and bringing the scientific revolution to its culmination. Isaac Newton's *Principia* in 1687 synthesized the results of what can now be seen to have been a century of collaborative effort, and serves to signalize a new era. Newton crowned the long endeavor to see the heavenly bodies as parts of a wonderful piece of clockwork.

The achievements of ancient Greece in the field of science had now been unmistakably transcended and outmoded. The authority of both the ancient and the medieval worlds was overthrown, and Western man was fully persuaded that he must rely on his own resources in the future. Religion had come to a low ebb after generations of fanaticism, persecution and war; now it was in a weak position for meeting the challenge of the new thought. The end of the 18th century sees in any case the decisive moment in the secularization of European society and culture. The apostles of the new movement had long been claiming that there was a scientific method which could be adapted to all realms of inquiry, including human studies—history, politics and comparative religion, for example. The foundations of what has been called the age of reason had now been laid.

At the same time society itself was changing rapidly, and man could see it changing, see it as no longer static but dynamic. There began to emerge a different picture of the process of things in time, a picture of history as the embodiment of progress rather than of decline. The future now appeared to offer opening vistas and widening horizons. Man was coming to feel more capable of taking charge over his own destiny.

It was not merely man's tools, and not merely natural science, that had carried the story forward. The whole complex condition of society was involved, and movement was taking place on a wide front. The age of Newton sees the foundation of the Bank of England and the national debt, as well as the development of speculation that was to culminate in the South Sea Bubble. An economic order congenial to individualism meant that life was sprouting from multitudinous centers, initiatives were being taken at a thousand points and ingenuity was in constant exercise through the pressure of need or the assurance that it would have its reward. The case is illustrated in 17th-century England by the famous "projectors"—financial promoters busy devising schemes for making money. They slide easily into reformers making plans for female education or a socialistic order or a better form of government.



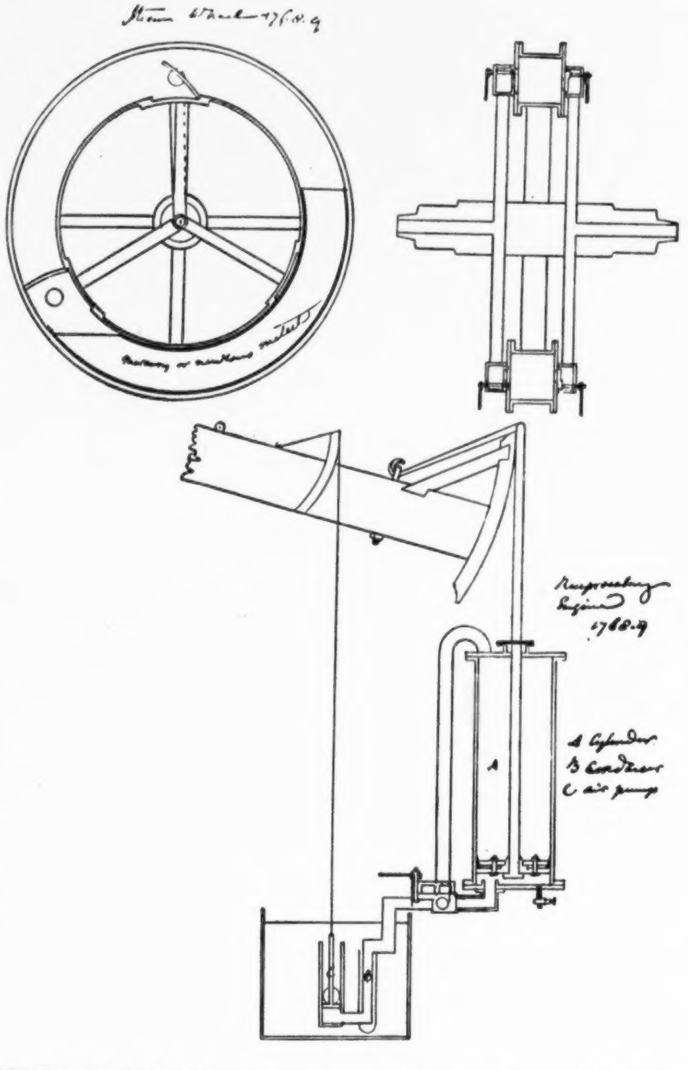
STRENGTH OF A BEAM was one of the problems in which Galileo demonstrated the power of mathematical methods in science. Illustration is taken from his *Discorsi e dimostrazioni matematiche*, in which he described the "new sciences" of mechanics and motion.

The whole of Western society was in movement, science and technology, industry and agriculture, all helping to carry one another along. But one of the operations of society—war—had probably influenced the general course of things more than is usually recognized. War above all had made it impossible for a king to "live of his own," enabling his subjects to develop constitutional machinery, to insist on terms in return for a grant of money. Because of wars, kings were allied with advanced capitalistic developments from the closing centuries of the Middle Ages. The growing demands of governments in the extreme case of war tightened up the whole development of the state and produced the intensification of the idea of the state. The Bank of England and the national debt emerge during a conflict between England and France, which almost turned into a financial war and brought finance into the very structure of government. In the 17th century armies had been mounting in size, and the need for artillery and for vast numbers of uniforms had an important effect on the size of economic enterprises.

The popularity in England of the natural sciences was paralleled to a degree by an enthusiasm for antiquarian pursuits. In the later decades of the 17th century the scientific method began to affect the development of historical study. In turn, the preoccupation with the process of things in time seems to have had an influence upon scientists themselves. Perhaps the presiding scientific achievement in the next hundred years was the application of biology, geology and allied studies to the construction of a history of the physical universe. By the end of the period this branch of science had come almost to the edge of the Darwinian theory of evolution. For the rest, if there was further scientific "revolution" in the 18th century, it was in the field of chemistry. At the beginning of the period it had not been possible to isolate a gas or even to recognize clearly that different gases existed. In the last quarter of the century Lavoisier reshaped this whole branch of science; water, which had been regarded for thousands of years as an element, was now seen to be a compound of oxygen and hydrogen.

By this time England—the nation of shopkeepers—was surprising the world with developments in the industrial field. A class of men had emerged who were agile in intellect, capable of self-help and eager for novel enterprises. They often lacked the classical education of the time, and were in a sense cut off from their cultural inheritance; and they no longer had the passion to intervene in theological controversy. Science and craftsmanship, combined with the state of the market, enabled them, however, to indulge their zeal for gadgets, mechanical improvements and inventions.

A considerable minor literature of the time gives evidence of the widespread passion for the production of technical devices, a passion encouraged sometimes by the policy of the government. Between 1760 and 1785 more patents were taken out than in the preceding 60 years; and of the estimated total of 26,000 patents for the whole century, about half were crowded into the 15 years after 1785. In 1761 the Society for the Encouragement of the Arts, Manufactures and Commerce, established a few years earlier, offered a prize for an invention that would enable six threads to be spun by a single pair of hands. A few years later Hargreave's spinning jenny and Arkwright's water frame appeared. The first steam engine had emerged at the beginning of the century, but textile factories began by using water power. The change to steam both here and in the production of iron greatly intensified the



DETAILS OF STEAM ENGINE are reproduced from James Watt's patent of 1769. The change from water to steam power in textile factories intensified the industrial revolution.

industrial revolution that was to alter the landscape so profoundly in the 19th century.

The country was able to meet the needs of a rapidly expanding population, especially as industrial development was accompanied by an agrarian revolution—the birth of something like modern farming. Possibly as a result of a change in the prevalent type of rat, England ceased to suffer from the plague that had ravaged it for centuries. Advances in public-health techniques helped reduce the death rate, especially among infants. During the 18th century the English population rose from 5.5 to nine million. And people flocked to swell the growing industrial towns, as though assured that they were fleeing from something worse to something better.

Even in 1700 most Englishmen were still engaged in occupations of a primary nature, connected with farming, fishing, mining and so on. London had perhaps half a million inhabitants, but Bristol, which came next, may have had only 20,000. Very few towns had a population exceeding 10,000. Each country town had its miller, its brewer, its tanner and so on; each village had its baker, its blacksmith and its cobbler. Many of the people who were employed in industry—in the making of textiles, for example—carried on the work in their own homes with hand looms and spinning wheels; they supplemented their income by farming.

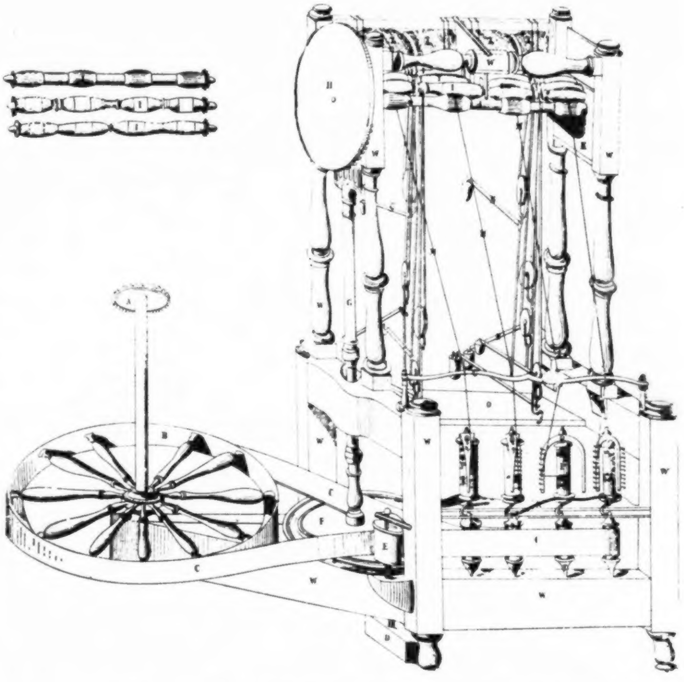
The coming of the factory system and the growth of towns represented an unprecedented transformation of life and of the human environment, besides speeding up the rate of all future change. This denser and more complicated world required more careful policing, more elaborate administration and a tremendous increase in the tasks of government. The mere growth and distribution of population, and the fresh disposition of forces that it produced within society, are fundamental factors in the history of the 19th century.

With gathering momentum came railways, the use of electricity, the internal-combustion engine and today the world of electronics and nuclear weapons. Science, so long an aid to the inventor, now seems itself to need the engineer and the industrial magnate. And all the elaborate apparatus of this technical civilization is easily communicable to every quarter of the globe. Our scientific-industrial revolution is a historical landmark for those peoples to whom Renaissance and Reformation have no relevance, since Christianity and Greek antiquity are not in their tradition. The material apparatus of our civilization is more communicable to other continents than are our more subtle and imponderable ideas.

Yet the humanism that has its roots so far back in our history has by no means lost its hold on the world. In the West, indeed, it now touches vastly wider classes of peoples than were able to read at all before the days of the industrial revolution. That revolution requires the spread of education, and at the same time provides the apparatus for it. The extraordinary speeding-up of communications and the increased mobility of life have themselves had colossal educative results. It was under the ancient order that the peasantry were sometimes felt to be like cows; John Wesley, although he held so firmly that the lowest classes were redeemable, himself described them with astonishing frequency as wild beasts. The new era has raised the stature of men, not lowered it, as some have imagined; and seems to require (or to produce) a more genuine kind of moral autonomy.

Great literature is perhaps more widely appreciated at the present day than ever in previous history. The rights and freedoms of man and the independence and self-respect of nations have never been more glorified than in our own century. And we have transmitted these ideals to other parts of the globe. The scientific-industrial revolution has operated to a great saving of life. At the same time it has provided a system which, where it has prevailed, has so far enabled the expanded population to live.

The vastness of populations and the character of the technical revolution itself have led, however, to certain dangers. The development of high-powered organization means that a colossal machine can now be put at the service of a possible dictatorship. It is not yet clear that the character of the resulting civilization will necessarily undermine the dictatorship and produce the re-establishment of what we call Western values. In this sense the elaborate nature of the system may come to undermine that wonderful individualism that gave it its start. At the same time, when nations



SPINNING FRAME, patented by Richard Arkwright in 1769, produced superior yarn. In his application the inventor said the machine would be of "great utility" to manufacturers and to the public "by employing a great number of poor people in working said machinery."

are ranged against one another, each may feel forced to go on elaborating and enlarging ever more terrible weapons, though no nation wants them or ever intends to use them. Weapons may then defeat their own ends, and man may find himself the slave of the machine.

The Western ideal of democracy is older than the scientific-industrial revolution, but it may eventually prove a necessary concomitant of that revolution, wherever the revolution may spread. At this point we simply do not know. There are certain things we cannot achieve without tools. But the tools in themselves do not necessarily determine our destiny.

The effect of the rise of physics in the age of Galileo and Newton, particularly on literature and religion, is discussed in this brief article.

16

How the Scientific Revolution of the Seventeenth Century Affected Other Branches of Thought

Basil Willey

1959

IN order to get a bird's-eye view of any century it is quite useful to imagine it as a stretch of country, or a landscape, which we are looking at from a great height, let us say from an aeroplane. If we view the seventeenth century in this way we shall be struck immediately by the great contrast between the scenery and even the climate of its earlier and that of its later years. At first we get mountain ranges, torrents, and all the picturesque interplay of alternating storm and brightness; then, further on, the land slopes down to a richly cultivated plain, broken for a while by outlying heights and spurs, but finally becoming level country, watered by broad rivers, adorned with parks and mansions, and lit up by steady sunshine. The mountains connect backwards with the central medieval Alps, and the plain leads forwards with little break into our own times. To drop the metaphor before it begins to be misleading, we may say that the seventeenth century was an age of transition, and although every century can be so described, the seventeenth deserves this label better than most, because it lies between the Middle Ages and the modern world. It witnessed one of the greatest changes which have ever taken place in men's ways of thinking about the world they live in.

I happen to be interested in literature, amongst other things, and when I turn to this century I cannot help noticing that it begins with Shakespeare and Donne, leads on to Milton, and ends with Dryden and Swift: that is to say, it begins with a literature full of passion, paradox, imagination, curiosity and complexity, and ends with one distinguished rather by clarity, precision, good sense and definiteness of statement. The end of the century is the beginning of what has been called the Age of Prose and Reason, and we may say that by then the qualities necessary for good prose had got the upper hand over those which produce the greatest kinds of poetry. But that is not all: we find the same sort of thing going on elsewhere. Take architecture, for example; you all know the style of building called Elizabethan or Jacobean—it is quaint and fanciful, sometimes rugged in outline, and richly ornamented with carving and decoration in which Gothic and classical ingredients are often mixed up together. Well, by the end of the century this has given place to the style of Christopher Wren and the so-called Queen Anne architects, which is plain, well proportioned, severe, and purely classical without Gothic trimmings. And here there is an important point to notice: it is true that the seventeenth century begins with a blend of medieval and modern elements, and ends with the triumph of the modern; but observe that in those days to be 'modern' often meant to be 'classical', that is, to imitate the Greeks and Romans. We call the age of Dryden, Pope and Addison the 'Augustan' Age, and the men of that time really felt that they were living in an epoch like that of the Emperor Augustus—an age of enlightenment, learning and true civilisation—and congratulated themselves on having escaped from the errors and superstitions of the dark and monkish Middle Ages. To write and build and think like the ancients meant that you were reasonable beings, cultivated and urbane—that you had abandoned the shadow of the cloister for the cheerful light of the market place or the coffee house. If you were a scientist (or 'natural philosopher') you had to begin, it is true, by rejecting many ancient theories, particularly those of Aristotle, but you knew all the while that by thinking independently and taking nothing on trust you were following the ancients in spirit though not in letter.

Or let us glance briefly at two other spheres of interest: politics and religion, beginning with politics. Here again you notice that the century begins with Cavalier and Roundhead and ends with Tory and Whig—that is to say, it begins with a division arousing the deepest passions and prejudices, not to be settled without bloodshed, and ends with the mere opposition of two political parties, differing in principle of course, but socially at one, and more ready to alternate peaceably with each other. The Hanoverians succeed the Stuarts, and what more need be said? The divine right of kings is little more heard of, and the scene is set for prosaic but peaceful development. Similarly in religion, the period opens with the long and bitter struggle between Puritan and Anglican, continuing through civil war, and accompanied by fanaticism, persecution and exile, and by the multiplication of hostile sects; it ends with the Toleration Act, and with the comparatively mild dispute between the Deists and their opponents as to whether Nature was not after all a clearer evidence of God than Scripture, and the conscience a safer guide than the creeds. In short, wherever you turn you find the same tale repeated in varying forms: the ghosts of history are being laid; darkness and tempest are yielding to the light of common day. Major issues have been settled or shelved, and men begin to think more about how to live together in concord and prosperity.

Merely to glance at this historical landscape is enough to make one seek some explanation of these changes. If the developments had conflicted with each other we might have put them down to a number of different causes, but since they all seem to be setting in one direction it is natural to suppose that they were all due to one common underlying cause. There are various ways of accounting for historical changes: some people believe, for instance, that economic causes are at the bottom of everything, and that the way men earn their living, and the way in which wealth is produced and distributed, determine how men think and write and worship. Others believe that ideas, rather than material conditions, are what control history, and that the important question to ask about any period is what men then believed to be true, what their philosophy and religion were like. There is something to be said on both sides, but we are concerned with a simpler question. We know that the greatest intellectual change in modern history was completed during the seventeenth century: was that change of such a kind as to explain all those parallel movements we have mentioned? Would it have helped or hindered that drift towards prose and reason, towards classicism, enlightenment and toleration? The great intellectual change was that known as the Scientific Revolution, and I think the answer to these questions is—Yes.

It is not for me to describe that revolution, or to discuss the great discoveries which produced it. My task is only to consider some of the effects it had upon men's thoughts, imaginations and feelings, and consequently upon their ways of expressing themselves. The discoveries—I am thinking mainly of the Copernican astronomy and the laws of motion as explored by Galileo and fully formulated by Newton—shocked men into realising that things were not as they had always seemed, and that the world they were living in was really quite different from what they had been taught to suppose. When the crystal spheres of the old world-picture were shattered, and the earth was shown to be one of many planets rolling through space, it was not everyone who greeted this revelation with enthusiasm as Giordano Bruno did. Many felt lost and confused, because the old picture had not only seemed obviously true to common sense, but was confirmed by Scripture and by Aristotle, and hallowed by the age-long approval of the Church. What Matthew Arnold said about the situation in the nineteenth century applies also to the seventeenth: religion had attached its emotion to certain supposed facts, and now the facts were failing it. You can hear this note of loss in Donne's well-known lines:

> And new philosophy calls all in doubt;
> The element of fire is quite put out;
> The sun is lost, and th' earth, and no man's wit
> Can well direct him where to look for it.

Not only 'the element of fire', but the very distinction between heaven and earth had vanished—the distinction, I mean, between the perfect and incorruptible celestial bodies from the moon upwards, and the imperfect and corruptible terrestrial bodies below it. New stars had appeared, which showed that the heavens could change, and the telescope revealed irregularities in the moon's surface—that is, the moon was not a perfect sphere, as a celestial body should be. So Sir Thomas Browne could write:

> 'While we look for incorruption in the heavens, we find they are but like the earth;—durable in their main bodies, alterable in their parts; whereof, besides comets and new stars, perspectives (i.e. telescopes) begin to tell tales, and the spots that wander about the sun, with Phaeton's favour, would make clear conviction.'

Naturally it took a long time for these new ideas to sink in, and Milton still treats the old and the new astronomies as equally acceptable alternatives. The Copernican scheme, however, was generally accepted by the second half of the century. By that time the laws governing the motion of bodies on earth had also been discovered, and finally it was revealed by Newton that the law whereby an apple falls to the ground is the very same as that which keeps the planets in their courses. The realisation of this vast unifying idea meant a complete re-focusing of men's ideas about God, Nature and Man, and the relationships between them. The whole cosmic movement, in the heavens and on earth, must now be ascribed no longer to a divine pressure acting through the Primum Mobile, and angelic intelligences controlling the spheres, but to a gravitational pull which could be mathematically calculated. The universe turned out to be a Great Machine, made up of material parts which all moved through space and time according to the strictest rules of mechanical causation. That is to say, since every effect in nature had a physical cause, no room or need was left for supernatural agencies, whether divine or diabolical; every phenomenon was explicable in terms of matter and motion, and could be mathematically accounted for or predicted. As Sir James Jeans has said: 'Only after much study did the great principle of causation emerge. In time it was found to dominate the whole of inanimate nature. . . . The final establishment of this law . . . was the triumph of the seventeenth century, the great century of Galileo and Newton.' It is true that mathematical physics had not yet conquered every field: even chemistry was not yet reduced to exactitude, and still less biology and psychology. But Newton said: 'Would that the rest of the phenomena of nature could be deduced by a like kind of reasoning from mechanical principles'—and he believed that they could and would.

I referred just now to some of the immediate effects of the 'New Philosophy' (as it was called); let me conclude by hinting at a few of its ultimate effects. First, it produced a distrust of all tradition, a determination to accept nothing as true merely on authority, but only after experiment and verification. You find Bacon rejecting the philosophy of the medieval Schoolmen, Browne writing a long exposure of popular errors and superstitions (such as the belief that a toad had a jewel in its head, or that an elephant had no joints in its legs), Descartes resolving to doubt everything—even his own senses—until he can come upon something clear and certain, which he finally finds in the fact of his own existence as a thinking being. Thus the chief intellectual task of the seventeenth century became the winnowing of truth from error, fact from fiction or fable. Gradually a sense of confidence, and even exhilaration, set in; the universe seemed no longer mysterious or frightening; everything in it was explicable and comprehensible. Comets and eclipses were no longer dreaded as portents of disaster; witchcraft was dismissed as an old wives' tale. This new feeling of security is expressed in Pope's epitaph on Newton:

How the Scientific Revolution of the Seventeenth Century Affected Other Branches of Thought

Nature and Nature's laws lay hid in night;
God said, *Let Newton be!* and all was light!

How did all this affect men's religious beliefs? The effect was very different from that of Darwinism on nineteenth-century religion. In the seventeenth century it was felt that science had produced a conclusive demonstration of God, by showing the evidence of His wisdom and power in the Creation. True, God came to be thought of rather as an abstract First Cause than as the personal, ever-present God of religion; the Great Machine implied the Great Mechanic, but after making the machine and setting it in motion God had, as it were, retired from active superintendence, and left it to run by its own laws without interference. But at a time when inherited religious sentiment was still very powerful, the idea that you could look up through Nature to Nature's God seemed to offer an escape from one of the worst legacies of the past—religious controversy and sectarian intolerance. Religion had been endangered by inner conflict; what could one believe, when the Churches were all at daggers drawn? Besides, the secular and rational temper brought in by the new science soon began to undermine the traditional foundations of belief. If nothing had ever happened which could not be explained by natural, physical causes, what about the supernatural and miraculous events recorded in the Bible? This was a disturbing thought, and even in the seventeenth century there were a few who began to doubt the literal truth of some of the biblical narratives. But it was reserved for the eighteenth century to make an open attack upon the miraculous elements in Christianity, and to compare the Old Testament Jehovah disparagingly with the 'Supreme Being' or 'First Cause' of philosophy. For the time, it was possible to feel that science was pious, because it was simply engaged in studying God's own handiwork, and because whatever it disclosed seemed a further proof of His almighty skill as designer of the universe. Addison exactly expressed this feeling when he wrote:

The spacious firmament on high,
With all the blue ethereal sky,

And spangled heavens, a shining frame,
Their great Original proclaim.
Th' unwearied Sun from day to day
Does his Creator's power display;
And publishes to every land
The work of an Almighty hand.

Science also gave direct access to God, whereas Church and creed involved you in endless uncertainties and difficulties.

However, some problems and doubts arose to disturb the prevailing optimism. If the universe was a material mechanism, how could Man be fitted into it?—Man, who had always been supposed to have a free will and an immortal soul? Could it be that those were illusions after all? Not many faced up to this, though Hobbes did say that the soul was only a function of the body, and denied the freedom of the will. What was more immediately serious, especially for poetry and religion, was the new tendency to discount all the products of the imagination, and all spiritual insight, as false or fictitious. Everything that was real could be described by mathematical physics as matter in motion, and whatever could not be so described was either unreal or else had not yet been truly explained. Poets and priests had deceived us long enough with vain imaginings; it was now time for the scientists and philosophers to take over, and speak to us, as Sprat says the Royal Society required its members to do, in a 'naked, natural' style, bringing all things as close as possible to the 'mathematical plainness'. Poets might rave, and priests might try to mystify us, but sensible men would ignore them, preferring good sense, and sober, prosaic demonstration. It was said at the time that philosophy (which then included what we call science) had cut the throat of poetry. This does not mean that no more good poetry could then be produced: after all, Dryden and Pope were both excellent poets. But when all has been said they do lack visionary power: their merits are those of their age—sense, wit, brilliance, incisiveness and point. It is worth noticing that when the Romantic movement began a hundred years later, several of the leading poets attacked science for having killed the universe and turned man into a reasoning machine. But no such thoughts worried the men of the Augustan Age; their prevailing feeling was satisfaction at living in a world that was rational through and through, a world that had been explained favourably, explained piously, and explained by an Englishman. The modern belief in progress takes its rise at this time; formerly it had been thought that perfection lay in antiquity, and that subsequent history was one long decline. But now that Bacon, Boyle, Newton and Locke had arisen, who could deny that the ancients had been far surpassed? Man could now hope to control his environment as never before, and who could say what triumphs might not lie ahead? Even if we feel that the victory of science was then won at the expense of some of man's finer faculties, we can freely admit that it brought with it many good gifts as well —tolerance, reasonableness, release from fear and superstition—and we can pardon, and even envy, that age for its temporary self-satisfaction.

Maxwell, the developer of electromagnetic theory (Unit 4), wrote light verse. The reference in the first line of the second poem is to the members of the British Association for the Advancement of Science.

17

Rigid Body (Sings)
Report on Tait's Lecture on Force:— B.A. 1876

James Clerk Maxwell

1876, earlier

IN MEMORY OF EDWARD WILSON,

Who repented of what was in his mind to write after section.

Rigid Body (sings).

GIN a body meet a body
  Flyin' through the air,
Gin a body hit a body,
  Will it fly ? and where ?
Ilka impact has its measure,
  Ne'er a ane hae I,
Yet a' the lads they measure me,
  Or, at least, they try.

Gin a body meet a body
  Altogether free,
How they travel afterwards
  We do not always see.
Ilka problem has its method
  By analytics high ;
For me, I ken na ane o' them,
  But what the waur am I ?

Report on Tait's Lecture on Force:—B.A., 1876.

Ye British Asses, who expect to hear
Ever some new thing,
I've nothing new to tell, but what, I fear,
May be a true thing.
For Tait comes with his plummet and his line,
Quick to detect your
Old bosh new dressed in what you call a fine
Popular lecture.

Whence comes that most peculiar smattering,
Heard in our section?
Pure nonsense, to a scientific swing
Drilled to perfection?
That small word "Force," they make[1] a barber's block,
Ready to put on
Meanings most strange and various, fit to shock
Pupils of Newton.

Ancient and foreign ignorance they throw
Into the bargain;
The shade of Leibnitz[2] mutters from below
Horrible jargon.
The phrases of last century in this
Linger to play tricks—
Vis Viva and *Vis Mortua* and *Vis*
Acceleratrix:—

Those long-nebbed words that to our text books still
Cling by their titles,
And from them creep, as entozoa will,
Into our vitals.
But see! Tait writes in lucid symbols clear
One small equation;
And Force becomes of Energy a mere
Space-variation.

Force, then, is Force, but mark you! not a thing,
Only a Vector;
Thy barbèd arrows now have lost their sting,
Impotent spectre!
Thy reign, O Force! is over. Now no more
Heed we thine action;
Repulsion leaves us where we were before,
So does attraction.

Rigid Body (Sings)
Report on Tait's Lecture on Force:— B.A. 1876

Both Action and Reaction now are gone.
Just ere they vanished,
Stress joined their hands in peace, and made them one :
Then they were banished.
The Universe is free from pole to pole,
Free from all forces.
Rejoice ! ye stars—like blessed gods ye roll
On in your courses.

No more the arrows of the Wrangler race,
Piercing shall wound you.
Forces no more, those symbols of disgrace,
Dare to surround you :
But those whose statements baffle all attacks,
Safe by evasion,—
Whose definitions, like a nose of wax,
Suit each occasion,—

Whose unreflected rainbow far surpassed
All our inventions,
Whose very energy appears at last
Scant of dimensions :—

Are these the gods in whom ye put your trust,
Lordlings and ladies ?
The hidden[1] potency of cosmic dust
Drives them to Hades.

While you, brave Tait ! who know so well the way
Forces to scatter,
Calmly await the slow but sure decay,
Even of Matter.

In tracing the relation of science to other parts of modern life, Jacob Bronowski interviews an artist, Eduardo Paolozzi, an architect, Eero Saarinen, a physicist, Abdus Salam, and a writer, Lawrence Durrell.

18

The Vision of Our Age

J. Bronowski

1964

This book began at the birth of a child, and traced its development until it enters 'the gateway to imagination and reason'. This is the stage when the child can manipulate objects in thought as well as with its hands: when it can make images of them. The child has little knowledge yet, in the ordinary sense of the word; but it has the mental equipment to learn and create knowledge. Once a child can make images, it can also reason, and build for itself a coherent picture of the world that is more than separate bundles of sense impressions.

We have just seen that when a child enters 'the gateway to imagination', it leaves all animals behind. Before it learns to make images, a young human develops in much the same way as a young animal. Children and animals alike have to learn to co-ordinate their various senses and to recognise objects. But after that, animals fall behind. They have no power of imagination. That is, they cannot carry images in the mind; and without imagery, without an inner language, they cannot manipulate ideas.

The theme of imagination runs through this book. We have examined some of the great achievements of science and seen that they are imaginative ideas. Science does not merely plod on like a surveyor, laboriously mapping a stretch of country, square mile by square mile. Of course nature must be surveyed, and very laborious that is at times; but the survey is not the end. The great moments in science come when men of imagination sit down and think about the findings—when they recreate the landscape of nature under the survey.

Science must be solidly grounded in fact and in experiment. But a blind search for experimental facts is not enough; it could never have discovered the theory of relativity. Science is a way of looking at things, an insight, a vision. And the theories of science are the underlying patterns that this way of looking at the world reveals. Many of the patterns are unexpected even at the simplest beginnings. (For example, common sense would not even have expected to find that stars and human beings are put together from the same basic building bricks of matter.) And the more unexpected the pattern, the greater the feat of imagination that is needed to see it for the first time.

What place have these imaginative ideas of science in our daily thoughts? Science and technology have transformed the physical world we live in; but have they yet had much effect on thought? Many people even dislike the ideas of science, and feel that they are abstract and mechanical. They reject science because they fear that it is in some way inhuman.

This book shows that science is as much a creation of the human imagination as art is. Science and art are not opposites; they spring from the same human impulses. In this last chapter, we shall examine their relations to one another, in the past and today. In particular, we shall see how both enter and combine into the way man in the twentieth century sees the world: the vision of our age. For this purpose, we shall include personal statements about their own work by an artist, an architect, a scientist, and a writer.

The artist is the sculptor Eduardo Paolozzi. The group of pictures show him in his studio, then one of his sculptures being cast in the foundry, then one of his finished sculptures called *San Sebastian*—with a jet engine standing in the background—and finally another recent work.

This is what Eduardo Paolozzi had to say about his work and the world for which it is made.

'I am a sculptor, which means that I make images. As a sculptor I was taught at the Slade the classical idea of being an artist. The best one could do would be to emulate Victorian ideals and to work in a studio executing portraits or monuments.

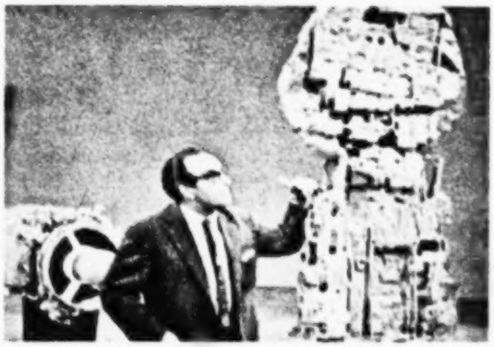
'But there has been a rejection now of the classical idea of tracing art out of art, which is in a way a sort of death process leading to the provincial gallery, with the atmosphere of the death-watch beetle—a gilt-edged, sure-thing idea of art.

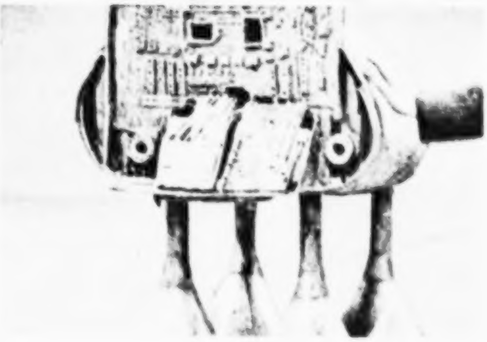
'In this century we have found a new kind of freedom—an opening up of what is possible to the artist as well as to the scientist. So I don't make copies of conventional works of art. I'm not working for Aunt Maud; I'm trying to do things which have a meaning for us living today. So I work with objects which are casual and natural today, that is, mechanisms and throwaway objects. To me they are beautiful, as my children are beautiful, though in a different way. I think they are different definitions of beauty.

'I haven't got any desire to make a sculpture of my children; but a wheel, a jet engine, a bit of a machine is beautiful, if one chooses to see it in that way. It's even more beautiful if one can improve it, by incorporating it in one's iconography. For instance, something like the jet engine is an exciting image if you're a sculptor. I think it can quite fairly sit in the mind as an art image as much as an Assyrian wine jar. I think it's a beautifully logical image, in the sense that anything in its delicate structure, with its high precision standards, has got a reason, almost in a way like human anatomy.

'My *San Sebastian* was a sort of God I made out of my own necessity: a very beautiful young man being killed by arrows, which has a great deal of symbolism in it. I think this is a good thing for young artists to identify themselves with, in a way that doing the Madonna and Child may not be a thing they can identify themselves with. It has two legs, which are decorated columns, it has a rather open, symbolical square torso, with disguised, warped, twisted, mechanic elements. Then the final element is a sort of drum with a space cut in the middle.

'What I feel about using the human diagram is that it points up in a more specific way the relationship between man and technology. There isn't any point in having a good idea in sculpture unless there is some kind of plastic or formal organisation. So I don't reproduce the jet engine, I transform it. And I use the wheel a lot in my sculpture as a symbol,



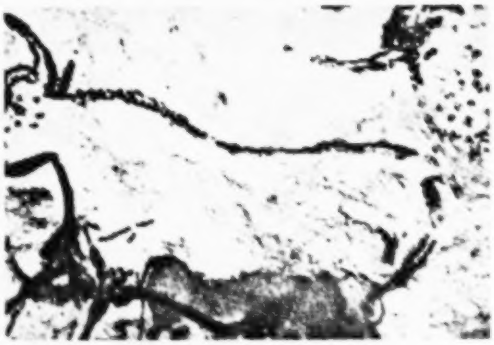


as a quickly read symbol, of the man-made object. This also refers back to my crude peasant idea of science, which is that the wheel gives the idea of man being able to get off the ground. The wheel to me is important, and the clock. I think this is very significant—I find the clock moving because I find modern science moving. I see it as a sort of heroic symbolism.

'In the last fifty years, science seems to be the outstanding leading direction, the most considerable direction that man has taken. It is trying continually to go beyond what was possible till that very moment. I think there is a possibility in what I call, crudely, higher science, a tremendous possibility of man being free. And I think it can give me a certain kind of moral strength, in the sense that art can move into a similar category of freedom. In my sculpture I am trying to speak for the way people are freeing themselves from traditional ideas. I'm a sculptor and so I put these ideas into images. If I do this well they'll be heroic images, ones that will survive and ones which other ages will recognise. Image making gives me the sense of freedom in a way that nothing else can.'

A word to which Paolozzi returns several times is 'free'. He feels that science frees man, from his conventions, from the restrictions of his environment, from his own fears and self-doubts. If this is true, then man has gained this growing freedom by imagination: in science, by imagining things that have not yet happened. Paolozzi wants to communicate the same sense of growing freedom in the images of his sculpture. He wants people to feel that they are heroic images.

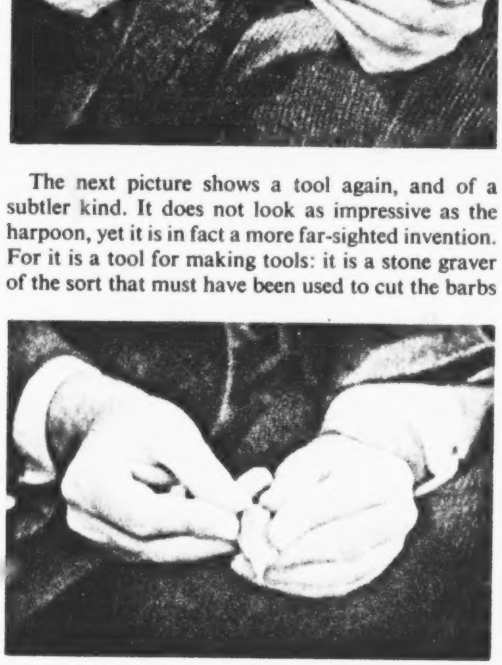
Science and art are both imaginative activities, and they present two sides of the imagination. The two sides have often tried and often failed to come together, in the past and in recent time. This chapter itself, and this book, is an attempt to help bring them together. Paolozzi's work is also an attempt to bring them together, in a different language. He uses the everyday products of technology (the stamped shapes in the first picture, for example) as the raw material of his art, because they seem to him as natural and expressive in modern civilisation as the human body itself.



It is interesting to look at the two sides of the human imagination in an earlier civilisation. We have evidence for them, long even before writing was invented. These paintings, in the caves of Lascaux in southern France, are at least twenty thousand years old. They are the most famous and the finest examples of art from the Stone Age. The word 'art' is not out of place, and yet it is most unlikely that these pictures were created in the same spirit as classical art. The caves of Lascaux were not a Stone Age art gallery that people came to visit. Art of this kind was an integral part of the civilisation of Stone Age man.

The Lascaux paintings are a product of one side of the imagination of the men who lived twenty thousand years ago. This picture shows a product of the other side of their imagination. It is a tool: a harpoon, cut from bone. It has barbs, like a modern fish hook, to stop it from being pulled out when it lodges in an animal.

The next picture shows a tool again, and of a subtler kind. It does not look as impressive as the harpoon, yet it is in fact a more far-sighted invention. For it is a tool for making tools: it is a stone graver of the sort that must have been used to cut the barbs in the harpoon. The men who invented this were able to think beyond the immediate needs of the day—killing an animal, cutting it up, scraping its hide. When they invented a tool for making tools (today we should call that a machine-tool) they took a new step of the imagination.



What is the link between paintings on the wall of a cave, and primitive tools made of bone and flint? Separated as we are by twenty thousand years from the men who created both, we can only speculate. But we are surely right in speculating that the paintings served some purpose other than mere decoration. Look at another Lascaux painting. It represents three bulls and (probably) a boar. A bull is being struck by a spear with barbs—a spear like the one that we have seen. This is plainly a hunting scene. Many of the other cave paintings show similar scenes. The painters were constantly preoccupied with hunting. This is why most authorities agree that the paintings were some kind of magic, and were intended to help the hunter to dominate the animal before the hunt started.

Unhappily, 'magic' is one of those words ('instinct' is another) that does not really explain anything. It merely says that we do not know the explanation. What kind of magic were the painters making? What did they feel they were doing for the hunters? How did they think that they were helping them to dominate the hunted animal?

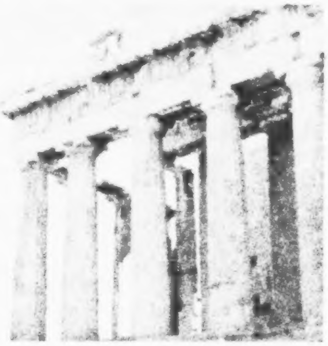
Here I will give my personal view. I think that the paintings helped the men who painted them, and the men who lived in the caves with them, to conquer their fear of the hunted animal. A bull was (and is) a dangerous beast, and out in the open there would not be much time to think about him. By drawing him you become familiar with him, get used to the idea of meeting and hunting him, and imagine ways in which he can be outwitted. The close-up makes the bull familiar to you; and the familiar is never as frightening as the unknown.

It is not far-fetched here to draw an analogy with modern methods of training. Consider, for example, the training of spacemen. They have to face a frightening situation, in which what they fear is simply the unfamiliar and unknown. They will not survive if they panic; they will do the wrong thing. So a long and life-like training programme is designed to make them familiar in advance with every situation that they are likely to encounter. The spaceman's training is more than a matter of simply learning to press the right buttons. It is also a psychological preparation for the unknown.

I believe that the Stone Age cave paintings were also a psychological preparation for the unknown.

They helped the Stone Age hunters to dominate their psychological environment, just as flint and bone tools helped them to dominate their physical environment. That is the connecting link between the two. Both are tools, that is, instruments which man uses to free himself and to overcome the limitations of nature. It was Benjamin Franklin who first defined man as 'the tool-making animal'. He was right, and the tools are mental as well as physical.

We move forward now many thousand years, to a time and place where the two sides of the human imagination worked more closely together than ever before, and perhaps ever since. The pictures on the right come from Athens of the fifth century B.C. The men who built this city had suddenly burst out of the confines of the cave and come into the light of freedom. Their civilisation recognised that man's most powerful tool in the command of nature is the human mind. The Greeks named their city, and the great temple of the Parthenon in it, after the goddess of wisdom, Athene. Light and reason, logic and imagination together dominated their civilisation.



Greek architecture, for example, has a strong mathematical basis, yet it never appears stiff and mechanical. Look at the Parthenon, as perfect a creation in architecture as man has made; and it is dominated by a precise sense of numbers. Numbers had a mystical significance for the Greeks (Pythagoras made them almost into a religion) and this expressed itself in all they did.

The Parthenon has 8 columns along the front and 17 along each side. That to the Greeks was the ideal proportion. The number of columns along each side of a temple should be twice the number along the front, plus one more. No Greek architect would have built otherwise.

Numbers that are perfect squares seemed to the Greeks equally fascinating and beautiful. The Parthenon is 4 units wide and 9 units long; for 4 is the square of 2, and 9 is the square of 3—the two smallest squares. The ratio of height to width along the front of the building is also 4 to 9; and so is the ratio of the thickness of the columns to the distance between them.

Yet all this arithmetic is not a dead ritual. The Greeks found it exciting because they found it in natural objects. To them, it expressed the mystery of nature, her inner structure. Numbers were a key to the way the world is put together: this was the belief that inspired their science and their art together.

So the Parthenon is nowhere merely a set of mathematical relations. The architect is guided by the numbers, but he is never hidebound by them.

His plan begins with arithmetic, but after that the architect himself has taken command of the building, and has given it freedom, lightness and rhythm. For example, the end columns are closer to their neighbours than are the other columns; and the end columns are also a little thicker. This is to make the building compact, to make it seem to look inwards at the corners. And all the columns lean slightly inwards, in order to give the eye (and therefore the building) a feeling of upward movement and of lightness.

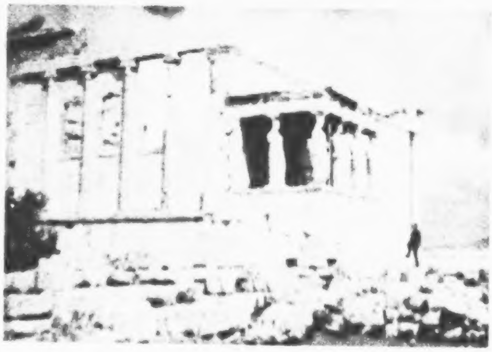
The pictures on the right are of the Erictheum. It stands close to the Parthenon, but is less famous. Perhaps that is because the Erictheum is less monumental, more slender, more delicate in its whole conception. Yet the mathematics is still there. The porch of the Erictheum, for instance, is designed on the 'golden section'. That is, the canopy has the same proportion to the base as the base has to the human figures which support the canopy. The golden section was a mathematical relation which was based on nature: on the proportions of the human body.

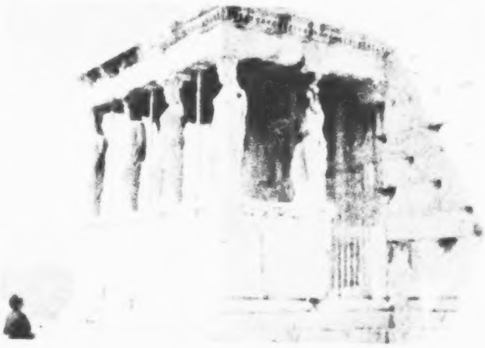
The human figures which support the canopy are made to seem in movement; two rest on the right foot, two on the left. Everywhere in the Erictheum there is the feeling of movement. The different levels of the building are joined together with suppleness and rhythm. This is what the Erictheum expresses in architecture: an almost musical sense of rhythm. And this reminds us that Pythagoras prided himself, rightly, on having discovered the mathematical structure of the musical scale.

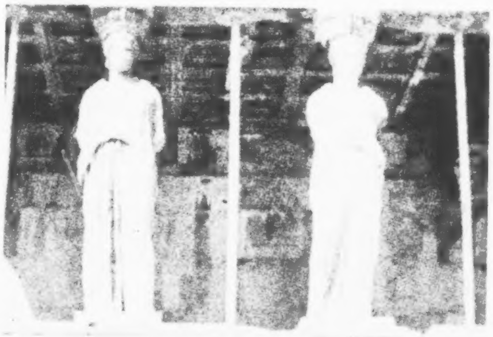
The fusion of the mathematical order with the human, of reason with imagination, was the triumph of Greek civilisation. The artists accepted the mathematics, and the mathematicians did not resent the architects imposing their individuality on the mathematical framework. It was a civilisation which expressed itself in the way things were put together—buildings, ideas, society itself. Greek architecture survives to illustrate this, perhaps better than any other record.

All architecture must begin with technical efficiency. Walls have to stand up, roofs have to keep the rain out. So an architect can never be unpractical, as can a painter or a sculptor. He cannot be content with the mere look of the thing. The side of the human imagination which made the Stone Age tools cannot be left out. But a bad architect can play it down, and can take the practical for granted, as a painter takes his canvas for granted.

The strength of the best architecture today is that it does not despise the practical purposes of buildings. It does not hide the structure and function under merely elegant decoration. Structure and function in modern buildings play the same fundamental part as numbers in Greek architecture. They form







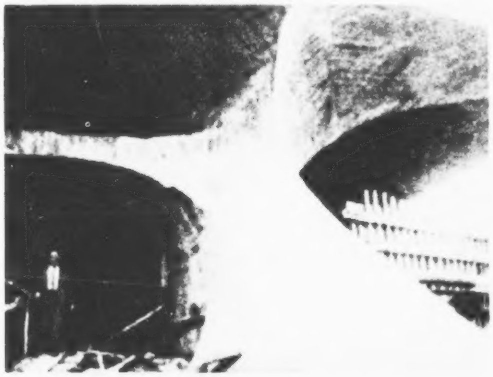
the framework on which the architect imposes his individual imagination. And he does not pretend that the framework is not there.

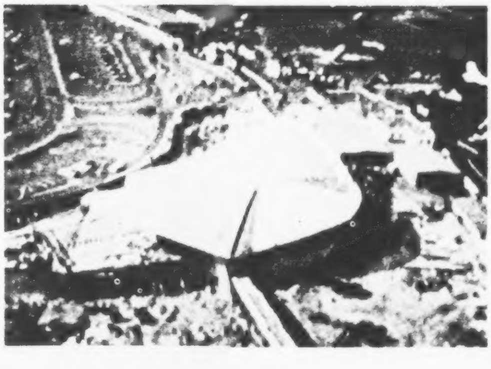
Our next personal statement comes from a famous architect, Eero Saarinen. He was born in Finland but built most of his great buildings in America. The pictures below show the building that he did not live to finish, the TWA Air Terminal at Idlewild Airport in New York. The lines of the building are very dramatic, and the form is consciously mathematical and aerodynamic. The question is: Is the bold, flying shape necessary, or is it a romantic artifice without a true function? I discussed this with Eero Saarinen during the building, and this is how he replied.

'To really answer your question, I would have to go a little bit back, and talk philosophically about architecture. As you know, we all, in architecture, have been working in this modern style, and certain principles have grown up within it. The basic principles are really three. There is the functional part. There is the structural part, honestly expressing the structure of the building. And the third thing is that the building must be an expression of our time. In other words, the technology of our time must be expressed in a building.

'Now those are the principles that we are all agreed on—the principles that one might have said ten years ago were the only principles. I think since that time more thought and maybe some more principles have grown up. I would say one of these additional principles, one which I believe in, is that where buildings have a truly significant purpose they should also express that purpose.'

Function and purpose were not the same thing in Saarinen's mind. The TWA Air Terminal has a clear function: to handle passengers into and out of aeroplanes. But for Saarinen, it also had a deeper purpose: from here people were to fly, and he wanted to give them the sense of freedom and adventure which flying has for earth-bound men. The vaulted shapes of the building were well-conceived as structures, but they were meant to be more: their aerodynamic and birdlike look was to express what Saarinen called the purpose—the sense of going off to fly. And the long spurs reaching out from the building show that it is not something self-contained, an end-point. They suggest entering the building and leaving it, which is of course what the passengers do.





Eero Saarinen went on:

'The last thing that I've become convinced of, and I'm not the only one, there are many others, is that once you've set the design, it must create an architectural unity. The idea of the barrel vaults making the roof of the Air Terminal building is carried through in all the details, even the furnishings.

'Basically architecture is an art, though it is halfway between an art and a science. In a way it straddles the two. I think to a large degree the motivating force in the designing of architecture comes from the arts side. If you ask, Are these curves and everything derived from mathematics? the answer is No. They are sympathetic with the forces within the vaults, which is mathematical, but there are so many choices which one has, and these really come from the aesthetic side.

'To me architecture is terribly important because it is really an expression of the whole age. After we're dead and gone, we're going to be judged by our architecture, by the cities we leave behind us, just as other times have been. What man does with architecture in his own time gives him belief in himself and in the whole period. Architecture is not just a servant of society, in a sense it's a leader of society.'

Architecture straddles art and science. That statement is true of the Greek architecture of two thousand years ago as well as of the architecture of today. In this, the Greek imagination is close to our own. The Greeks were preoccupied with the idea of structure; and we have seen in this book that the idea of structure is also central to modern science. Like the Greeks, the modern scientist is always looking at the way things are put together, the bones beneath the skin. How often in this book have we used such phrases as 'the architecture of matter'!

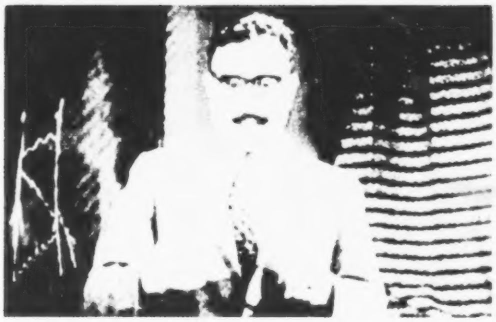
For example, the Greeks invented the idea of the atom as the smallest unit of matter from which everything in the world is built. Plato thought there were five kinds of atom, and he pictured them as the five regular solids of geometry. The first four were the atoms of the four kinds of matter: earth, air, fire, and water; one of these is shown in the first picture below. The fifth was the universe itself, the unity of the other four—we still call it the quintessence; it is shown, as Plato imagined it, in the second picture.

This conception is fantastic, and the atoms it pictures have no relation to the facts. And yet the fanciful pictures are a first attempt to solve, imaginatively, the same problems of structure and behaviour that the modern physicist faces. The Greek conception and the modern theories about atoms are both attempts to explain the bewildering complexity of the observable world in terms of an underlying, unifying order. Greek scientific theories are now only of historical interest. Yet before the Greeks, no one had thought about the world in this way at all. Without them, there would have been no modern science. It was the Greeks who first formulated the problems that modern science tries to answer.

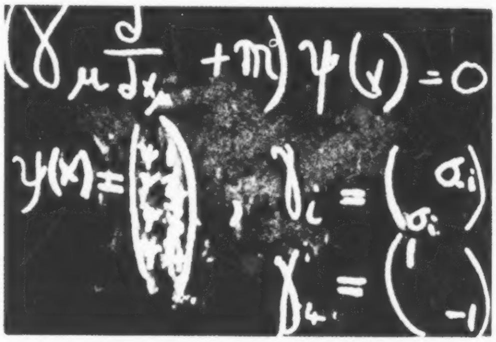
Our third personal statement comes from a physicist: Professor Abdus Salam, of the Imperial College of Science in London. He describes some modern ideas about atoms. They are a long way from Plato's regular solids; yet, as Professor Salam points out, that is where they started. Here is what Salam said.

'I am a theoretical physicist, and we theoretical physicists are engaged on the following problem. We would like to understand the entire complexity of inanimate matter in terms of as few fundamental concepts as possible. This is not a new quest. It's the quest which humanity has had from the beginning of time—the Greeks were engaged on it. They conceived of all matter as being made up of fire, water, earth and air. The Arabs had their ideas about it too. Scientists have been worried about this all through the centuries. The nearest man came to solving this problem was in 1931 when, through the work done in the Cavendish Laboratory in Cambridge, we believed that all matter consisted of just two particles—electrons and protons—and all forces of nature were essentially of two kinds, the gravitational force and the electrical force.

'Now we know that this view of 1931 was erroneous. Since that time the number of particles has increased to thirty, and the number of elementary forces to four. In addition to the electrical and gravitational forces, we now believe that there are two other types of force, both nuclear—one extremely strong, and the other extremely weak. And the task we are engaged on is to try to reduce this seeming complexity to something which is simple and elementary.



'Now the type of magic which we use in order to solve our problem is first to rely on the language which we use throwing up ideas of its own. The language which we use in our subject is the language of mathematics, and the best example of the language throwing up ideas is the work of Dirac in 1928. He started with the idea that he would like to combine the theory of relativity and the theory of quantum mechanics. He proceeded to do this by writing a mathematical equation, which he solved. And to his astonishment, and to everyone's astonishment, it was found that this equation described not only the particles—electrons and protons—which Dirac had designed the equation for, but also particles of so-called anti-matter—anti-electrons, anti-protons.


$(\gamma_\mu \frac{\partial}{\partial x_\mu} + m)\psi(x) = 0$
$\psi(x) =$
$\gamma_i = (\alpha_i, \sigma_i)$
$\gamma_4 = (1, -1)$

'So in one stroke Dirac had increased the number of particles to twice the number. There are the particles of matter, there are the particles of anti-matter. In a sense, of course, this produces simplicity too, because when I speak of thirty particles, really fifteen of them are particles and fifteen of them are anti-particles. The power of mathematics as a language that suggests and leads you on to something, which we in theoretical physics are very familiar with, reminds me of the association of ideas which follows when possibly a great poet is composing poetry. He has a certain rhyme, and the rhyme itself suggests the next idea, and so on. That is one type of way in which invention comes about.

'The second type of idea which we use to solve our problems is the idea of making a physical picture. A very good illustration is the work of the Japanese physicist Yukawa in 1935. Yukawa started to ponder on the problem of the attractive force between two protons, and he started with the following picture. Suppose there are two cricketers, who have a cricket ball, and they decide to exchange the ball. One throws the ball and the other catches it, perhaps. Suppose they want to go on exchanging the ball, to and fro, between them. Then the fact that they must go on exchanging the ball means that they must keep within a certain distance of each other.

'The result is the following picture. If one proton emits something which is captured by the second one, and the second one emits something which is captured by the first one, then the fact that they have to capture, emit, re-absorb constantly means that they will remain within a certain distance of each other. And someone who cannot see this intermediate object, this ball, the object we call the meson, will think that these two protons have an attractive force between them. This was Yukawa's way of explaining the attractive force between two elementary particles.

'The result of Yukawa's work was that he predicted that there do exist such particles which play the role of intermediate objects. And he predicted that such particles would have a mass about three hundred times that of electrons. Yukawa made this prediction in 1935. In 1938 these particles were discovered, and we now firmly believe that the forces of nature, all forces of nature, are transmitted by this type of exchange of intermediary particles.

'Now so far I have been talking about our methods, but what is really important are our aims. Our aim in all this is to reduce the complexity of the thirty elementary particles and the four fundamental forces into something which is simple and beautiful. And to do this what we shall most certainly need is a break from the type of ideas which I have expressed—a complete break from the past, and a new and audacious idea of the type which Einstein had at the beginning of this century. An idea of this type comes perhaps once in a century, but that is the sort of thing which will be needed before this complexity is reduced to something simple.'

The ideas put forward by Salam are vivid. But more than the specific ideas, we are interested here in his description of science itself. For him, science is the attempt to find in the complexity of nature something which is simple and beautiful. This is quite different from the usual view that science collects facts and uses them to make machines and gadgets. Salam sees science as a truly imaginative activity, with a poetic language of its own. This is an arresting point that Salam made: that the mathematics in science is a poetic language, because it spontaneously throws up new images, new ideas.

Science can learn from the language of poetry, and literature can learn from the language of science. Here we bring in our fourth contributor. He is Lawrence Durrell, who wrote the four famous books which make up *The Alexandria Quartet*. In this fourfold novel, space and time are treated in an unusual way, and Durrell began by talking about this.

'I was hunting for a form which I thought might deliver us from the serial novel, and in playing around with the notions of relativity it seemed to me that if Einstein were right some very curious by-products of his idea would emerge. For example, that truth was no longer absolute, as it was to the Victorians, but was very provisional and very much subject to the observer's view.

'And while I felt that many writers had been questing around to find a new form, I think they hadn't succeeded. I don't know of course, I've only read deeply in French. There may well be Russian or German novels which express this far better than I have.

'But they hadn't expressed what I think Einstein would call the 'discontinuity' of our existence, in the sense that we no longer live (if his reality is right) serially, historically, from youth to middle age, to death; but in every second of our lives is packed, in capsule form, a sort of summation of the whole. That's one of the by-products of relativity that I got.

'In questing around for a means of actually presenting this in such an unfamiliar form as a novel, I borrowed a sort of analogy, perhaps falsely, from the movie camera. I'd been working with one, and it seemed to me that when the camera traverses across a field and does a pan shot, it's a historic shot in the sense that it goes from A to B to C to D. And if it starts with a fingernail and backtracks until you get a whole battlefield, that seemed to me a spatialisation. It was rooted in the time sequence that it was spatialising; it was still enlarging spatially.

'I tried to mix these two elements together, and see what would happen to ordinary human characters in what is after all a perfectly old-fashioned type of novel—an ordinary novel, only not serial. I found, somewhat to my own surprise, that I was getting a kind of stereoscopic narrative, and getting a kind of stereophonic notion of character. This excited me so much that I finished it and tried to add the dimension of time by moving the whole thing forward—you know, "read our next issue"—five years later. And there it is, ready for the critics to play with.'

Here are Lawrence Durrell's answers to some questions about his work:

Q. You said that you got from relativity the feeling that truth was provisional, or at least depended very much on the observer.

A. Well, the analogy again is the observer's position in time and space. It's so to speak the fulcrum out of which his observation grows, and in that sense it is not an absolute view, it's provisional. The subject matter is conditioned by the observer's point of view.

Q. You're really making the point that the most important thing that relativity says is that there are no absolutes?

A. I was saying, most important for me. I think that any average person who's not a mathematician would assume that that was probably the most important part of it.

Q. I want to recall another phrase that you've just used. You said of your novel that 'after all it's a perfectly old-fashioned novel'. Now I don't feel that. I feel that your novel could have been written at no time but in the twentieth century.

A. Yes, in that sense certainly. But I was trying to distinguish between the form which, I believe, if it has come off at all, is original, and the content. When I was building the form I did something new. I said to myself, this is the shape: there are three sides of space, one of time. How do I shift this notion into such an unusual domain as the novel? And at the back of my mind I wondered whether we in the novel couldn't escape our obsession with time only.

Q. Your dimensions, as it were, deepen out each character as a recession in space. You show how different he becomes when he is seen by someone else from another point.

A. Stereoscopically, you see.

Q. I want to ask you a crucial question. Do you feel that the kind of inspiration that you've drawn from the scientific idea of relativity here is valid for everyone? That we can all in some way make a culture which combines science and the arts?

A. Surely a balanced culture must do that. And I think all the big cultures of the past have never made very rigid distinctions. Also I think that the very great artists, the sort of universal men, Goethe for example, are as much scientists as artists. When Goethe wasn't writing poetry he was nourishing himself on science.

Q. We can't expect everybody to be a Goethe, so how are we going to unify what is obviously different—the sense of what the artist is doing and the sense of what the scientist is doing?

A. I think by understanding that in every generation the creative part of the population feels called upon to try and attack this mysterious riddle of what we're doing, and to give some account of themselves. We're up against a dualism, because some people have more intelligence and less emotion, and vice versa. So the sort of account they give may suddenly come out in a big poem like Dante's, or it may come out in a Newtonian concept. In other words, the palm isn't equally given in each generation. But I feel that they're linked hand in hand in this attack on what the meaning of it all is.

The meaning of it all: the meaning of the pattern of nature, and of man's place in nature. Durrell's quest is also Salam's quest, and Saarinen's, and Paolozzi's. It is the quest of every man, whether scientist or artist or man in the street.

The driving force in man is the search for freedom from the limitations which nature has imposed. Man, unlike the animals, is able to free himself. The first crude attempts were already made by Stone Age man with his tools and paintings. Now, twenty thousand years later, we are still struggling for freedom. We try to reach it by understanding the meaning of things. Our age tries to see things from the inside, and to find the structure, the architecture which underlies the surface appearance of things. We command nature by understanding her logic.

Our age has found some unexpected turns in the logic of nature. How atoms evolve, much like living species. How living things code and pass on their pattern of life, much like a machine. How the rigorous laws of nature are averaged from the million uncertainties of atoms and individuals. How time itself is an averaging and a disordering, a steady loss of the exceptional.

How life opposes time by constantly re-creating the exceptional. And how profoundly our ideas of so safe and absolute a concept as time once seemed to be can be changed by the vision of one man, who saw and proved that time is relative.

Above all, our age has shown how these ideas, and all human ideas, are created by one human gift: imagination. We leave the animals behind because they have no language of images. Imagination is the gift by which man creates a vision of the world.

We in the twentieth century have a vision which unifies not only the physical world but the world of living things and the world of the mind. We have a much greater sense of person than any other age. We are more free than our ancestors from the limitations both of our physical and of our psychological environments.

We are persons in our own right as no-one was before us. It is not only that we can travel into space and under the oceans. Nor is it only that psychology has made us more at home with ourselves. It is a real sense of unity with nature. We see nature not as a thing but as a process, profound and beautiful; and we see it from the inside. We belong to it. This above all is what science has given us: the vision of our age.

Attempts to predict when things will happen, and what will be available in the future, are as fascinating as they are risky. Arthur Clarke, a science-fiction writer and scientist, has had unusual success in predicting future technical advances.

19

Chart of the Future

Arthur C. Clarke

1962

THE PAST

DATE	TRANSPORTATION	COMMUNICATION INFORMATION	MATERIALS MANUFACTURING	BIOLOGY CHEMISTRY	PHYSICS
1800			Steam engines	Inorganic chemistry	Atomic theory
	Locomotive				
		Camera Babbage calculator		Urea synthesized	
	Steamship	Telegraph	Machine tools		Spectroscope
1850					Conservation of energy
			Electricity	Organic chemistry	
		Telephone Phonograph Office machines			Electromagnetism Evolution
	Automobile		Diesel engine		
1900			Gasoline engine	Dyes	X-rays
	Airplane				Electron
		Vacuum tube	Mass production	Genetics Vitamins	Radioactivity
			Nitrogen fixation	Plastics	
1910					
					Isotopes
		Radio			
					Quantum theory
1920				Chromosomes Genes	
					Relativity Atomic structure
1930				Language of bees Hormones	
					Indeterminacy
		TV			Wave mechanics Neutron
1940	Jet Rocket Helicopter	Radar			
		Tape recorders			
		Electronic computers	Magnesium from sea	Synthetics Antibiotics	Uranium fission Accelerators
		Cybernetics	Atomic energy	Silicones	Radio astronomy
1950		Transistor	Automation		
	Satellite GEM	Maser Laser	Fusion bomb	Tranquilizers	I.C.Y. Parity overthrown

Chart of the Future

NOW

DATE	TRANSPORTATION	COMMUNICATION INFORMATION	MATERIALS MANUFACTURING	BIOLOGY CHEMISTRY	PHYSICS
1960	Spaceship	Communication satellite		Protein structure	Nucleon structure
THE FUTURE					
	Space lab				
1970	Lunar landing	Translating machines			
	Nuclear rocket		Efficient electric storage	Cetacean languages	
1980	Planetary landings				
					Gravity waves
		Personal radio		Exobiology	
1990			Fusion power	Cyborgs	
		Artificial intelligence			
2000	Colonizing planets		"Wireless" energy		
				Time, perception enhancement	Sub-nuclear structure
		Global library	Sea mining		
2010					
	Earth probes	Telesensory devices			
			Weather control		
2020		Logical languages			Nuclear catalysts
	Interstellar probes			Control of heredity	
		Robots			
2030			Space mining		
		Contact with extra-terrestrials		Bioengineering	
2040				Intelligent animals	
			Transmutation		
2050	Gravity control			Suspended animation	
		Memory playback			
	"Space drive"		Planetary engineering		
2060		Mechanical educator			
					Space, time distortion
		Coding of artifacts		Artificial life	
2070					
			Climate control		
	Near-light speeds				
2080		Machine intelligence exceeds man's			
	Interstellar flight				
2090	Matter transmitter		Replicator		
	Meeting with extra-terrestrials	World brain		Immortality	
2100			Astronomical engineering		

ARTISTS AND WRITERS

PERCY WILLIAMS BRIDGMAN

P. W. Bridgman was born in Cambridge, Massachusetts in 1882, entered Harvard in 1900, received his Ph.D. in physics there in 1908, and in 1913 became Professor. He retired in 1954, and died in 1961. Bridgman's experimental work was in high-pressure physics, for which he received the Nobel Prize in 1946. He has made important contributions to philosophy of science; for example, we owe him first detailed articulation of the concept of operational definition.

JACOB BRONOWSKI

Jacob Bronowski, who received his Ph.D. from Cambridge University in 1933, is now a Fellow of the Salk Institute of Biological Studies in California. He has served as Director of General Process Development for the National Coal Board of England, as the Science Deputy to the British Chiefs of Staff, and as head of the Projects Division of UNESCO. In 1953 he was Carnegie Visiting Professor at the Massachusetts Institute of Technology.

HERBERT BUTTERFIELD

Herbert Butterfield is Professor of Modern History at the University of Cambridge. He graduated from Cambridge and was elected a Fellow of Peterhouse at the same institution in 1923. He became Master of Peterhouse in 1955 and vice chancellor of the University in 1959. His writings include books on the history of religion, international affairs, and the history of science.

ALEXANDER CALDER

Alexander Calder, famous American sculptor and inventor of the mobile, was born in Pennsylvania in 1898. Intending to become an engineer, Calder entered the Stevens Institute of Technology, graduating in 1919. But by 1926 he had already published his first book (Animal Sketches) and presented his first exhibition of paintings. A visit with the Dutch artist Piet Mondrian in 1930 oriented him toward abstraction, and the next year he produced the first "stabiles," and in 1932, the first "mobiles." In these mobiles, Calder was able to incorporate motion into sculpture.

ARTHUR C. CLARKE

Arthur C. Clarke, British scientist and writer, is a Fellow of the Royal Astronomical Society. During World War II he served as technical officer in charge of the first aircraft ground-controlled approach project. He has won the Kalinga Prize, given by UNESCO for the popularization of science. The feasibility of many of the current space developments was perceived and outlined by Clarke in the 1930's. His science fiction novels include Childhoods End and The City and the Stars.

WILLIAM KINGDON CLIFFORD

W. K. Clifford was born in Exeter, England in 1845. He entered Trinity College, Cambridge in 1863, and graduated second in his class in mathematics. In 1871 he was appointed Professor of Applied Mathematics at University College, London, and three years later was named a Fellow of the Royal Society. He died in 1897. His mathematical ideas were often very far ahead of the times.

Clifford was an accomplished speaker, and was particularly adept at the popular exposition of abstruse doctrines. He was influenced by the writings of Charles Darwin and Herbert Spencer, and was concerned with the implications of science for ethics.

RICHARD PHILLIPS FEYNMAN

Richard Feynman was born in New York in 1918, and graduated from the Massachusetts Institute of Technology in 1939. He received his doctorate in theoretical physics from Princeton in 1942, and worked at Los Alamos during the Second World War. From 1945 to 1951 he taught at Cornell, and since 1951 has been Tolman Professor of Physics at the California Institute of Technology. Professor Feynman received the Albert Einstein Award in 1954, and in 1965 was named a Foreign Member of the Royal Society. In 1966 he was awarded the Nobel Prize in Physics, which he shared with Shinichero Tomonaga and Julian Schwinger, for work in quantum field theory.

JAMES BASIL GERHART

James Gerhart is Professor of Physics at the University of Washington in Seattle. Before coming to Washington, he taught at Princeton, where he received his Ph.D. in 1954. Professor Gerhart's specialty is nuclear physics.

J. B. S. HALDANE

J. B. S. Haldane is a British geneticist who serves as Professor of Biometry at University College, London. He pioneered in the application of mathematics to the study of natural selection and to other aspects of evolutionary theory. His broad grounding in mathematics, physics, and biology has enabled him to make uniquely insightful contributions in many different areas.

GERALD HOLTON

Gerald Holton received his early education in Vienna, at Oxford, and at Wesleyan University, Connecticut. He has been at Harvard University since receiving his Ph.D. degree in physics there in 1948; he is Professor of Physics, teaching courses in physics as well as in the history of science. He was the founding editor of the quarterly *Daedalus*. Professor Holton's experimental research is on the properties of matter under high pressure. He is a co-director of Harvard Project Physics.

FRED HOYLE

Fred Hoyle is an English theoretical astronomer, born in Yorkshire in 1915. Now Professor of Astronomy at Cambridge University, he is perhaps best known for one of the major theories on the structure of the universe, the steady state theory. Hoyle is well known for his scientific writing, and his success in elucidating recondite matters for the layman.

GYORGY KEPES

Gyorgy Kepes was born in 1906 in Selyp, Hungary. From 1930 to 1936 he worked in Berlin and London on film, stage, and exhibition design. In 1937 he came to the United States to head the Light and Color Department at the Institute of Design in Chicago. Since 1946 he has been Professor of Visual Design at the Massachusetts Institute of Technology. He has written *The New Landscape in Art and Science*, *Language of Vision*, and edited several books, including those in *the Vision + Value* series. Professor Kepes is one of the major painters; his work is included in the permanent collections of many museums.

PAUL KIRKPATRICK

Born in South Dakota, Paul Kirkpatrick received his doctorate in physics in 1923. Before reaching Stanford in 1931, he taught in China and Hawaii. At Stanford, he was named Professor of Physics in 1937, and became Professor Emeritus in 1959. Professor Kirkpatrick has served as education advisor with the U.S. Overseas Mission to the Philippines, and with the UNESCO mission to India.

JAMES CLERK MAXWELL

James Clerk Maxwell was born in Edinburgh, of a prominent Scottish family, in 1831. He graduated second in his class in mathematics at Cambridge, and was appointed to a professorship at Aberdeen in 1856. Shortly thereafter he demonstrated that Saturn's rings were composed of small particles. Next, Maxwell considered the mechanics of gases, and helped develop the kinetic theory. Maxwell's crowning achievement was his mathematical formulation of the laws of electricity and magnetism. He showed that electricity and magnetism were related, and proposed that light was one form of electromagnetic radiation. In 1871, Maxwell was appointed first Professor of Experimental Physics at Cambridge. He died eight years later, his life cut short by cancer.

HERBERT MATTER

Herbert Matter was born in Engelberg, Switzerland, on April 25, 1907. After graduating from college, he studied painting at L'Ecole des Beaux Arts in Geneva, and under Fernand Leger in Paris. In 1936 he came to the United States to work as a free-lance photographer for Harper's Bazaar, Vogue, and others. Since then he has won First Prize in the Polio Poster Design Competition, and worked on the design program for the New Haven Railroad and for the Boston and Main Railroad, among many other projects. Presently he is the Design and Advertising Consultant for Knoll Association, Inc., New York, graphic consultant to the Solomon R. Guggenheim Museum, New York, and Professor of Photography and Graphic Design at Yale University.

RUDI HANS NUSSBAUM

Rudi Nussbaum was born in Germany in 1922, he received his Ph.D. from the University of Amsterdam in experimental physics in 1954. Since then he has served as UNESCO research fellow at the Nuclear Physics Laboratory in Liverpool, as a senior fellow at CERN in Geneva, and is now Professor of Physics at Portland State College.

GEORGE POLYA

George Polya was born in Budapest in 1887. He studied in Vienna, Göttingen, and Budapest, where he received his doctorate in mathematics in 1912. He has taught in Zurich, and in this country at Brown University, Smith College, and Stanford University, where he served as Professor of Mathematics from 1946 to 1953. He is now Professor Emeritus.

JACOPO DA PONTORMO, (JACOPO CARRUCCI)

Born at Pontormo, Italy, May 24,1494, Jacopo Carrucci, later to be known as Jacopo da Pontormo, was one of the first of the Florentine Mannerists. Apprenticed to Leonardo da Vinci and later to Albertinelli and Piero di Cosimo, Pontormo broke away from the classical High Renaissance style. His altarpiece (still in the church of S. Michele Visdomini, Florence) exemplifies his intensely emotional style, in contrast to the traditional harmonically balanced style. Pontormo was buried in Florence on January 2, 1557.

DUANE H. D. ROLLER

Duane H. D. Roller is a graduate of Columbia University, where he received the A.B. in History of Science in 1941, of Purdue University (M.S. in Experimental Physics in 1949), and of Harvard University, where he was awarded the Ph.D. in History of Science and Learning in 1954. Since 1954 Dr. Roller has been at the University of Oklahoma, where he is McCasland Professor of the History of Science.

W. W. SAWYER

W. W. Sawyer was born in England in 1911. He attended Highgate School and St. John's College, Cambridge, where he specialized in the mathematics in England, New Zealand, and in the United States at the University of Illinois. At present he is Professor of Mathematics at Wesleyan University, where he edits the *Mathematics Student Journal*.

C. L. STONG

C. L. Stong was born in 1902 in Douds, Iowa. He attended the University of Minnesota, the Amour Institute in Chicago, and the University of Michigan (Detroit). For thirty years he was an engineer with Western Electric. Mr. Stong has also been involved in movie production, and in the early 1920's he was a stunt flier. Since 1948 he has been a contributor to *Scientific American*; his column, the Amateur Scientist, appears monthly.

WARREN WEAVER

Warren Weaver received his PH.D. in mathematics and physics from the University of Wisconsin in 1921, and remained at his alma mater, becoming Professor of Mathematics and Chairman of the Department in 1928. In 1932 he was appointed Director of Natural Sciences at the Rockefeller Foundation, and in 1955 was named Vice-president. He later was associated with the Sloan-Kettering Institute, and since 1959 has been with the Alfred P. Sloan Foundation. He is the recipient of the Arches of Science Award given by the Pacific Science Center of Seattle "for outstanding contributions to the improved public understanding of science."

BASIL WILLEY

Basil Willey was born in 1897 and later attended Peterhouse College, Cambridge, where he read history and English. From 1946 to 1964 he served as King Edward VII Professor of English Literature at Cambridge. In 1958 he was selected as President of Pembroke College, Cambridge, and is now an Honorary Fellow. His published works include many studies in English and the history of ideas.

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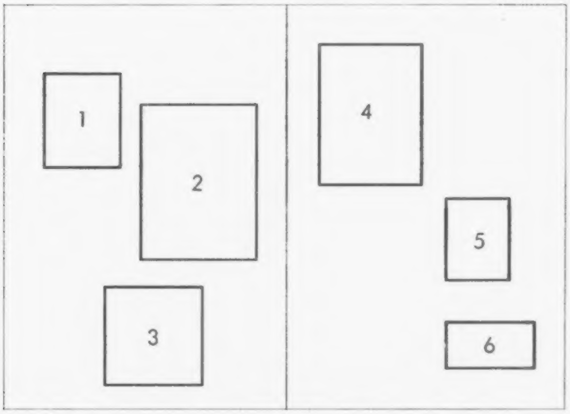
P. 110 Two photographs by Herbert Matter of Alexander Calder's "Hanging Mobile, 1936." Courtesy of the Museum of Modern Art, New York, from the collection of Mrs. Meric Callery, New York.

P. 126 Dr. Harold E. Edgerton, Massachusetts Institute of Technology, Cambridge.



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